# PROBLEM-SOLVING FRAMEWORK

## A discussion paper on teaching problem-solving


### Abstract

Problem-solving is a quintessential skill applicable to every sphere of our lives. This discussion paper proposes a problem-solving framework that is universal in nature and discusses how this framework can be used in teaching problem-solving. The discussion frames the problem-solving process with the focus on the problem's deliverable while addressing the inherent and specific conditions on the deliverable in a systematic way, using the dual principle of eyes-on-the-prize and just-in-time information.



Ana Duff

ana.duff@ontariotechu.ca


August 3, 2022

# Problem-solving framework – a discussion paper on teaching problem solving

## Contents

## Introduction

### Preamble

The intent of the discussion in this paper is to facilitate teaching problem-solving through a step-wise process of thinking and analysis. The discussion in this paper strictly applies to the process of solving problems where the full description of the problem, including a specific deliverable and the conditions it has to satisfy, are present and not under debate. Additionally, the discussion addresses only the process of arriving to a solution of a problem, not how to present a solution.

### The problem of solving a problem

When you get to the end of this sentence, stop and take a moment to consider the following question: what is the first thing one should do when asked to solve a problem?

Your answer likely started with the words: first understand the problem.

This is what most of us tell our students – start by "understanding the problem" – yet our students continue to struggle in this endeavour, to our continued chagrin. We explain that to understand the problem, one must understand the information given and look for ways of how it connects to the problem. Many students remain exasperated, feeling "they don't know where to start". Others dive in trying to find a way to connect the information provided in every which way, drawing on solutions to problems they were exposed to before, and seeing if they can somehow match those with the problem at hand. Most then fail at what they needed to do first and foremost: to identify the general context of the problem and the problem's deliverable. This is that elusive first step which all of us with enough life experience and education have eventually learned to take when facing a problem. We do it, yes, but often don't communicate it when presenting a solution, or we communicate it but only in half of a breath, chasing straight into the so-called information we are given and the strategy-building to solve the problem.

Yet, it is precisely in that bush that the rabbit lies – as one would say in my mother tongue.

To help our students not muddle through problem-solving process, but rather to approach it through disciplined, analytical, efficient pathways, we need to be less vague ("understand the problem") and more theoretically concrete ("consider this, then consider that"). This is truly reflective of the problem-solving framework those of us who are now good at solving problems have learned to work within innately. We must refrain from neatly presenting a solution and instead demonstrate the hard, messy work of arriving to one.

### Before we start, some definitions

What is a problem? Many definitions abound, but we can consider that of Webster's New World College Dictionary[1] which says that, in American English usage, a problem is:

- a question proposed for solution or consideration
- a question, matter, situation, or person that is perplexing or difficult
- a proposition requiring solution by mathematical operations, constructions, etc.

[1] Webster's New World College Dictionary, 4th Edition. Copyright © 2010 by Houghton Mifflin Harcourt. All rights reserved.

In other words, for the purposes of this discussion, the term *problem* will mean the totality of the problem description, which consists of explicit or implicit statement of one or more deliverables (or what is ultimately sought), conditions imposed on the deliverable(s), and background information for the purpose of context setting.

Each problem comes within a context, defined by Webster's New World College Dictionary as:

- the parts of a sentence, paragraph, discourse, etc. immediately next to or surrounding a specified word or passage and determining its exact meaning
- the whole situation, background, or environment relevant to a particular event, personality, creation, etc.

We can also refer to *general context* and *specific context*. The level of detail providing the context will determine whether we are referring to the first or the latter, from low to high level of detail, respectively.

Unless we are simply engaged in a philosophical banter, a problem will typically come with some task for us, demanding a deliverable. We will go with Wikipedia's definition[2] of a *deliverable* as:

- a tangible or intangible good or service produced as a result of a project that is intended to be delivered to a customer (either internal or external). A deliverable could be a report, a document, a software product, a server upgrade or any other building block of an overall project. A deliverable may be composed of multiple smaller deliverables. It may be either an outcome to be achieved or an output to be provided.

Consider, for example, a math problem discussing a change in average weekly wage in a particular industry in Canada. The task to calculate the percent change in the average weekly wage and to interpret the result will have two deliverables:

- the number representing the percent change in the average weekly wage, and
- the explanation of the meaning of this number within the context of the problem.

### The process of problem-solving pitfalls

One of the common problem-solving method students are given is that of GRASP:

- Given: What is known about the problem? (making note of information provided)
- Required: What do we need to find out? (making note of the deliverable)
- Analysis: How will we find the solution? (making note of relationships)
- Solution: Find the solution. (making note of steps towards the deliverable)
- Paraphrase: What is the answer? (stating the deliverable)

This method is exemplary of the types of problem-solving methods that often result in confusion, frustration and, very often, incorrect solutions. The primary issue is that, by starting with the totality of information provided, we reduce the cognitive capacity[3] for

---

[2] https://en.wikipedia.org/wiki/Deliverable. Accessed 13 February 2022.

[3] For a discussion on cognitive capacity in relation to working memory and reasoning, see *Separating Cognitive Capacity from Knowledge: A New Hypothesis* by Halford, Cowan and Andrews, Trends Cogn Sci. 2007 June ; 11(6): 236–242. doi:10.1016/j.tics.2007.04.001

analysis because of information overload. Additionally, without first clearly identifying the general context and our target, our ability to contextualize and direct the information we are given is limited and results in inefficiencies and misdirection. Without having our target clearly identified, we are then often erroneously influenced by previous experiences, leading us to ignore the actual deliverable and resulting in wasted time and energy on an irrelevant result.

This is akin to first asking a person to consider the collection of screws, bolts, lumber pieces and screwdrivers and, only after they have looked at all of the pieces and tried to make sense of them, asking them to identify what they are asked to produce, which is a bookshelf.

Alternatively, we could tell them to first identify what their task is, which is to build a bookshelf, and then to consider the tools they have available to deliver that task.

In the first instance, in the absence of identified deliverable, the mind will start to create potential deliverables using the tools that are given. Maybe these tools are to build a desk, a chair, a bookshelf, maybe we are expected to take inventory of the tools, maybe to determine what is missing, or to determine how to package the items most efficiently… This results in wasting cognitive resources on creating pathways to non-relevant products, creates potential for confusion and distraction, and requires the brain to simultaneously juggle the possible different sets of relationships concerning all of the information provided when only a subset (or possibly none at all) actually relates to the problem deliverable. Not to mention the trap the mind is led into when information not relevant to the deliverable is in the mix.

### Humble mindset and the discipline of the mind

Unfortunately, throughout most of our elementary and high school education we are taught, by example and by demonstration, that problems are easy to solve if you just know your stuff. When asked, students will almost universally concur with a vision that one should be able to figure out how to solve a problem (or even simply "see" the answer right away) from reading it once. The impression is then that if one is not able to do that, they don't know their stuff or, more generally, they are not good at problem-solving.

This is borne out of continually and almost without exception being presented the final, neatly packaged solutions to problems given by a person (a teacher, a book author, etc.) who solved the problem prior to presenting its solution. Unbeknownst to the student, the presenter has worked on the problem, read it many times, and tried different strategies, failing and succeeding and even making errors before they finally get to the solution. Then they re-packaged that solution for their presentation, with all of the hard, fingers in the dirt, messy work completely out of the picture. They present the problem once to their audience, in the course of which they deliver this beautiful argument that magically ends in the answer. And the student, whether awed or just simply convinced by the argument, is still left with the big old question – how did the presenter know to start here, and then go there, then there, then there, building their argument to the final answer?

Compounding this issue is our tendency towards building the solution pathway in our brain and only selectively choosing parts of the argument to share. In this, we rely on (or imply that) the other components of the argument are self-evident when, in most cases, they are anything but and in fact form an integral part of the argument in its completeness.

When reading one of the early drafts and coming to this point in the paper, my neighbor Charmaine chuckled and then exclaimed:

> *Exactly! Like when my kid was being taught how to add using a number line, and they were told that to add 20 and 30 all they had to do is start at 20 and then jump to the right by ten then ten then ten, upon which they asked me – but how do I know that I have to jump ten then ten then ten?*

Of course, someone might say that's obvious. But is it? Why ten then ten then ten? Because 30 is made of a ten and a ten and another ten, which is clearly not an obvious fact if you are only learning how to add tens.

The learning process of how to solve problems, and how to teach problem-solving, requires a certain degree of humbleness – recognizing, accepting, and expecting that statements of facts and the connections between them are not self-evident unless they are self-evident to, say, a five-year old.

To those of us who are well-versed in the problem-solving process, no matter how trivial some argument seems to us, we need to be self-critical of that assessment of self-evidence. Is something truly *self-evident,* or is it rather simply *evident to us*, but not necessarily to others who do not share our knowledge, experience, or fluidity in the process of analytical reasoning?

In the same vein, when teaching and guiding our students through learning problem-solving, we need to make it understood that the expected flow of things is that of mess, uncertainty, and doubt. We need to demonstrate that slowly but diligently, through discipline of the mind, we can (but will not always) get ourselves on the path that will lead us to our deliverable. As teachers, to best teach problem-solving we must continually, openly, humbly, and loudly demonstrate this process with every problem we solve for or with our students.

In our practice of teaching, we have to be continually demonstrating that the problem-solving process, when done effectively and efficiently, boils down to a question-answer dance. This dance takes the form of an interplay of memory recall of facts, considerations of the conditions presented in the problem, and the analytical process of connecting the recalled and the given. It requires multiple reviews of the problem statement throughout the solution-building process, where each review is carried by a different objective.

*Where is this problem situated, what is this problem about? This problem is about… What is my task, what am I asked to deliver? I am asked to … What do I know in general about...? I know that … Which pieces of information do I have and which ones do I still need to determine? I am given that … … … Did I achieve the task I was given, do I have my deliverable? Yes, …*

## Problem-solving framework

### Where to start?

As discussed in the introduction, in any problem all details provided except for those of very general nature should come secondary to establishing our target. We have to ask ourselves – what is that we are asked to produce? What are our deliverables?

## Establishing the target

There are two components in establishing the target, the first a prerequisite to the second. They are the context and the deliverable(s).

The first one relates to getting a general sense of the setting of the problem – *general* being the operative word. Is this problem about painting airplanes or buying sunflowers? No specifics; we only need enough to "plant one's feet", to have a context for the problem's deliverable.

Most problem descriptions start with the background that includes relevant and non-relevant conditions describing the specific context. The problem descriptions will also have the task, or description of the deliverable, stated at the end or buried somewhere in the middle. Our first job is to identify the general setting and to trust that we don't have to capture all of the details right away. It takes the discipline of the mind, the acknowledgement of the need, and the comfort in knowing that one will be able to return to the problem description in order not to get pulled into the weeds.

Thus, the first problem review needs to skate over any and all details, to push aside the brain's need to zero in on the numbers, relationships or conditions presented. No matter how long or complex the problem description is, that first read-through must have a three to five-word answer to the following question:

***(1) What is the general setting?***

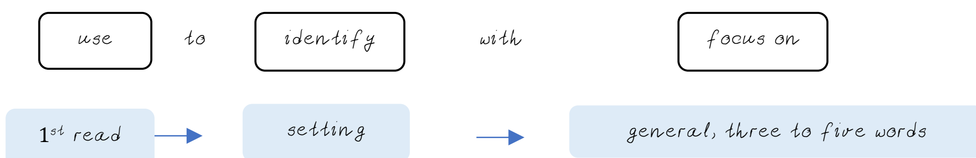



*Figure 1*

This question is arguably the hardest part of problem-solving framework that is the subject of this paper. This is not because it is hard to identify the general context of the problem at hand. Rather, it is because it is extremely difficult to not get drawn into the details and start spinning possibilities, overloading the cognitive space with not-yet-relevant data and relationships. This is the first test of the discipline of the mind.

The next, then, is identifying the deliverable. Though we know that getting the sense of the overall context is generally not difficult, identifying the deliverable can be less straight-forward. When helping our students learn efficient and effective problem-solving, we should draw on the fundamental connection to the language itself: to look for action verbs such as *find*, *determine*, *calculate*, *describe*, *interpret*, *create*, and *present*.

Here, too, we have to emphasize the need to focus on the target itself before considering the conditions that may be attached immediately in the statement of task, such as those following the words *that* or *if*.

Consider, for example, a problem where we identified that the general context is that of a change in salary. The problem description may contain a statement identifying the deliverable as follows:

$$\ldots\ \underbrace{Determine\ Mariam's\ salary}_{\text{relevant to identification of the deliverable}}\ if\ \underbrace{her\ salary\ increased\ by\ 1.37\%\ from\ last\ year}_{\text{not relevant to identification of the deliverable}}.\ \ldots$$

In the process of identifying the target, the part of the statement referring to the increase is irrelevant, especially the specific values. This part will only become relevant (and that only possibly) once we have considered what we know about the deliverable and how it can be achieved and, following that, once we are ready to consider the specific conditions imposed on our deliverable. The only part of the statement in the example above that is relevant in the problem deliverable identification is that our deliverable is the amount representing Mariam's salary. In other words, our guiding question throughout the problem will be: What is Mariam's salary?

In a case study, our deliverable might be some set of recommendations, and thus our guiding question while working on the solution to the problem will be: What should be the recommendations? The conditions, general and specific, will only become relevant (or irrelevant) once the target is identified. The same applies if the deliverable is a proof of a statement, a plan, a report, a painted airplane or something else. In other words, the question we must next answer is:

***(2) What is our deliverable?***

Then: Write it down, and concisely. Mark it visually so that your eyes can clearly identify it and thus the brain can be easily reminded, whenever needed, what your target is. As said in my mother tongue: write it so that it could be seen from an airplane.

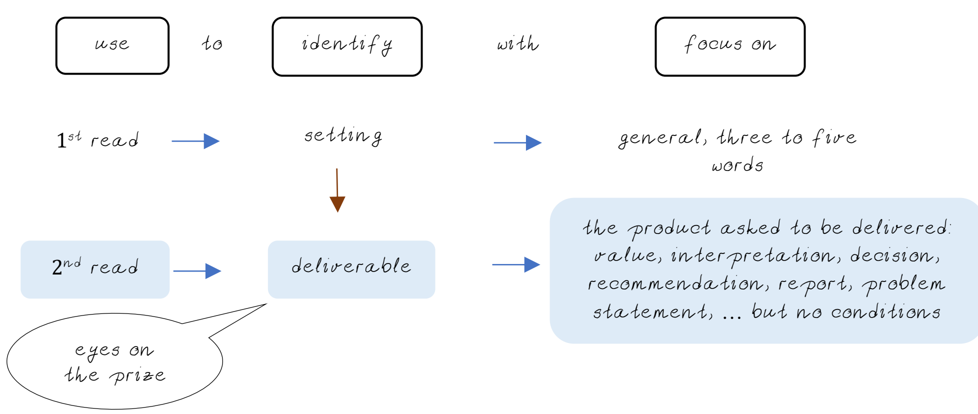



*Figure 2*

## Recall of and reflection on facts

Upon getting a sense of the general context and identifying the deliverable within that context, we next recall and reflect on general facts we know about the deliverable. Some will be relevant and some won't, but we should consider as many as we can within the context we identified. This will allow us to later consider the information or conditions imposed within the problem as each relates (or does not) to the objective of our task – delivering the deliverable.

The key here is again the discipline of the mind: to not rush into details provided in the problem description immediately after identifying our target. Except perhaps for the simplest of problems, the sheer number of connections we may be able to spin from the details provided will be enormous, cluttering the mind and making it difficult to not be led astray.

We should begin with the meaning of the words used in the description of the deliverable, i.e. definitions. This step is often skipped or brushed aside because an assumption is made that it is either unnecessary to spend time on specifying what the words involved mean or that everyone involved (including the problem poser and the problem solver) all agree on the meaning of those words. Yet, the definitions are most often the first stepping stone towards the identification of a solution pathway.

For example, even the ubiquitous statement such as *Solve the equation* requires the understanding of the meaning of those words before an attempt can be made to find the deliverable (the solution to the equation). In other words, we have to make a note that to *solve an equation* means that we are asked to *determine the value(s) of the unknown(s) involved in the equation which make the equation statement true*. This, in turn, leads us to a clearer understanding of what we are asked to deliver and the conditions imposed.

Following the recall of definitions related to the deliverable, we should consider general relationships involving the deliverable within the general context of the question.

For example, if the deliverable is the marginal revenue at 10,000 units, one would have to recall the meaning of the word *marginal*, the meaning of the word *revenue*, and within which relationships these terms, *revenue* and *marginal revenue*, could be considered. (Specifically, the recall and reflection may bring forth that the word *marginal* means the rate of change in output per unit of input, that *revenue* means amount of money gained through sales, that *revenue* can be calculated by multiplying the number of units sold by price per unit or as a difference of profit and cost, and that *marginal revenue* can be determined by calculating the value of the derivative of the revenue.)

Therefore, once we have a sense of the general context and have identified the deliverable, we have to answer the question:

***(3) What do I know in general about the deliverable?***

Then: Write what you recall down, and concisely. If a definition, list all conditions defining the term. If a relationship, use words rather than acronyms or variable names (as used in a formula) as much as possible. Putting them down will assist in creating a roadmap and will reduce the brain overload.

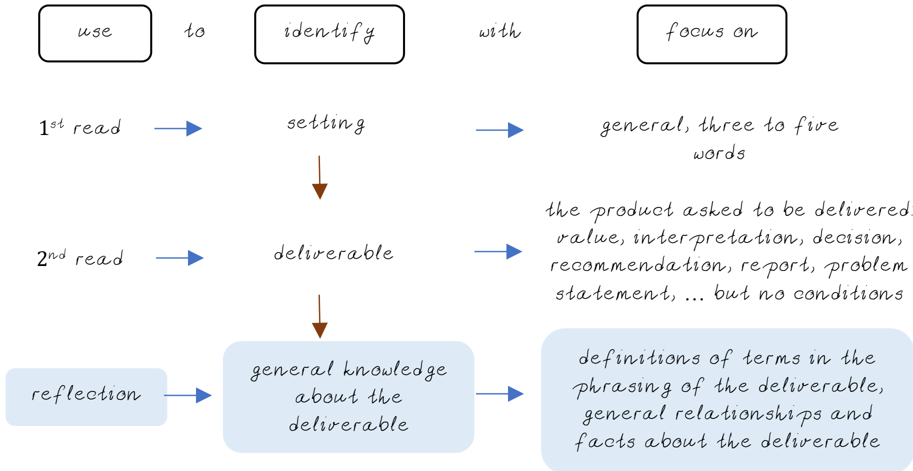



*Figure 3*

## And what is that I need for that?

Digging into our understanding of the deliverable as a general concept and the relationships involved will allow us to make our first steps towards building a solution strategy. Having that understanding, we can now start to identify how the parts that make up our deliverable are connected. This, in turn, will allow us to consider the details provided in the problem description as they directly relate, potentially relate or do not relate to the deliverable. It will also allow us to create potential, though typically very high-level, roadmaps: to get my deliverable, I need this, that, and the other thing, put together this way or maybe that way…

Ultimately, this step in the problem-solving process is the answer to that quintessential student question: *How did you know to start there?* Where to start does not begin with the information we are given. It begins by taking the general facts and relationships involving our deliverable and identifying how to rearrange or rephrase those facts and relationships so that we can use the specific context described in the problem to achieve our deliverable. In other words, our next step is to answer the question:

***(4) What information and tools do I need or could potentially use to determine the deliverable?***

Some answers to that question may come to mind quickly, especially if our recall of general facts and relationships contain statements such as: *the deliverable is made of/calculated by/determined from…*

However, some general relationships involving our deliverable may not describe it explicitly; rather, they may require a rearrangement of sort that results either in an equivalent or an inferential statement. This could include rephrasing of a statement by changing its grammatical structure or form, use of different vocabulary, or a rearrangement of a mathematical equation or inequality.

For example, by reviewing the following problem statement, we determined that the general setting is that of property taxes and budgets, and that the deliverable is the next year's, or new mill rate:

> *A school board is determining next year's operating budget and calculates that it needs an additional $5 million to be collected through property taxes. Properties in its municipality have an assessed value of $8.455 billion. The current mill rate for the school board is set at 6.1998. If the assessed property values are forecasted to rise by 3% next year, what mill rate should the school set?*[4]

In order to attempt finding a solution to this problem, we have to first ask ourselves what the meaning of the term *mill rate* is and recall relationships that involve that term. (In particular, mill rate is a rate applied to assessed value of a property to calculate a tax amount due, typically used in municipal tax collection for funding municipal services. We may also recall that property tax can be calculated by dividing the mill rate by 1,000 and then multiplying the result by the property value assessment. In this relationship, the mill rate is not described explicitly (i.e., we don't have a statement starting with *mill rate is…*). Rather, it is described implicitly using one known quantity (1,000) and two unknown quantities (tax amount and property value assessment). Using rules of algebra, we can restate this relationship as follows: *the mill rate is the property tax amount divided by the property value assessment, which is then multiplied by 1,000*. Now we have an explicit description of our deliverable and we understand which parts may help us build it, and in what way.)

Playing in a deliberate way with words (or symbols which, in reality, represent words) towards being able to describe our deliverable explicitly will allow us to create a pathway or pathways that show promise. In other words, if at all possible, we want to be able to start with a statement: *my deliverable is created from this and that, or from that and the other thing, in this way and that way, and so if I can determine this and that, or that and the other thing, I will be able to get my deliverable.*

[4] Adapted from J. Olivier (2018). *Business Math: A Step-by-Step Handbook*. Creative Commons License (CC BY-NC-SA)

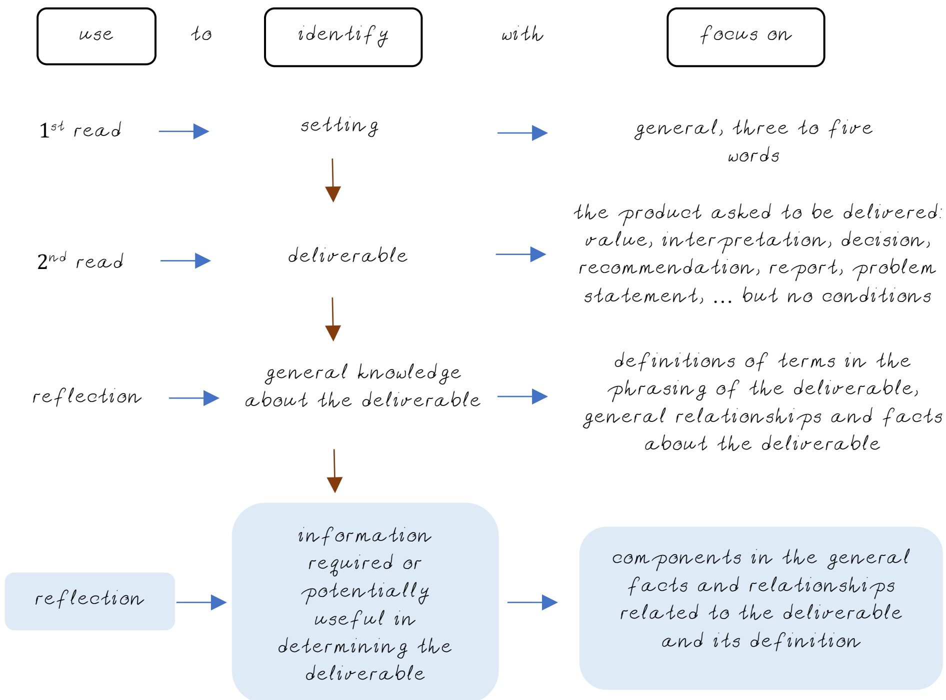



*Figure 4*

## Getting into the meat of things

### Assumption vs. information vs. condition

Rarely, or perhaps never, will our deliverable be unconditional. Even the consideration of the meaning of the words involved in the statement of the deliverable will often impose conditions, as would the general context and general conventions. A typical problem statement will also contain some specific conditions imposed on our deliverable.

We usually refer to both of these sets of conditions as assumptions. Due to the tendency of arguments that arise when the term *assumption* is used, in this paper we will avoid that term and will use the term *conditions*, with the assumption that the conditions are not arguable. In other words, we will assume that the argument over whether a particular condition stated in the problem description applies has already taken place, and that the resolution of that argument is that indeed it does.

For example, if one is asked to design a community school, and one of the conditions is that it has to be built on a hill, for the purpose of this discussion paper it is assumed that the question over why and whether the school should really be built on a hill has already been resolved and is moot.

We will also use the word *information* sparingly because, in the mind of the author, it can be considered as a special case of the word *condition*. More importantly, it is not conducive to creating an analytical pathway. We are more inclined to think of an information statement as a something that is just sort of there, to be considered in the generality of the problem, and not in terms of the action of making, implying, or drawing upon other statements. The term *condition*, on the other hand, draws us to the words such as *if*, *because, since, then, therefore,* *hence, we can conclude…* This, in turn, is more likely to help us in making connections to our deliverable, connecting what we do not know to what we know and are given.

### Relationships, equations, statements of equality and equivalent statements

Students who are novices to problem solving struggle with meaningfully connecting the conditions they are given to the deliverable they are tasked with. This is often a result of not-yet-well-developed skill of *playing with words*. Because problem descriptions are typically not created by the individual who is tasked with solving the problem, the choice of verbiage may not be conducive to finding relationships within the problem either in the way that the problem-solver thinks or in the way that the available tools work.

For example, rephrasing of statements may be needed in order to identify the conditions of equality, equivalency, and inference in terms relevant to tools and methods most applicable in the context of the problem.

Equality refers to statements such as "something is equal to another thing" whereas "equivalency" refers to statements such as "this statement is the same thing as saying this other statement". Inference refers to a sequence of statements that, in chain-like fashion, lead logically or otherwise, from one statement to another. Being clear and demonstrating to students that there is a distinction between the three, and how this distinction plays out in formulation of conditions and in the analytical process, is paramount in supporting our students in developing their problem-solving skills. For example,

- An equality (a single statement indicating equality):

  *the amount in the mutual fund is (equal to) twice the amount in stocks*

- An equivalency (two statements that mean the same thing):

  *they invested twice as much in the mutual fund as in stocks is the same thing as saying that their mutual fund investment is equal to two times their stock investment*

- An inference (truth of one statement implies truth of another statement):

  *they invested twice as much in the mutual fund as in stocks, therefore they invested more in the mutual fund than in stocks*

  *they invested more in the mutual fund than in stocks since their mutual fund investment is equal to two times their stock investment*

We can think of statements of equality (or, for that matter, inequality) as the building blocks in the creation of the solution pathway to the deliverable from the given, with equivalency and inference statements creating connections between (in)equalities.

Thus, an essential skill in problem-solving is being able to rephrase a given statement into an equivalent or inferential statement that is perhaps more suitable to the general relationships recalled through reflection or to the specific conditions stated in the problem. This is not something that is easy; it requires continual practice and is closely dependent on student's logical reasoning skills as well as language and literacy skills.

## Just-in-time consideration of relationships and facts

A well-known concept in business efficiency is the concept of *just-in-time delivery*. As defined by Investopedia[5], the "just-in-time (JIT) inventory system is a management strategy that aligns raw-material orders from suppliers directly with production schedules. Companies employ this inventory strategy to increase efficiency and decrease waste by receiving goods only as they need them for the production process, which reduces inventory costs. This method requires producers to forecast demand accurately."

As discussed earlier in the paper, there is a cost to juggling many details in what one could call our *thinking space*, the space of cognition in which we are aware of and actively engaged in considerations. When we hold onto a detail in our thinking space, this detail is tossed back and forth with other details akin to keeping balls in the air as if by a juggler. The more details, the less room in the cognitive space for both the processing and the storage. A detail held onto just in case and without a clear need for it at a specific moment in the analytical process reduces the efficiency of the mind as the mind works to build connections between the deliverable and the relevant conditions. It also potentially increases the processing waste created by considering solution pathways that are not productive.

Therefore, analogous to the just-in-time inventory concept in business, to increase analytic efficiency and to decrease cognitive waste when problem-solving one should employ the strategy of *just-in-time information.* In other words, we should, as much as possible, only consider relationships and facts that are directly related to the particular task at hand at a given stage in our solution-building process. We also have to keep in mind that what is directly related to the task at hand will change throughout the process as our tasks move from one to the next.

For example, at the start of the strategy exploration process, after we identified the deliverable, surmised our general knowledge of the deliverable, and identified what we will need to obtain it, our focus should turn to the conditions given in the problem. However, we have to be very discriminate at this stage. We must consider only those conditions that *directly* relate to the deliverable.

[5] Just-in-Time (JIT). https://www.investopedia.com/terms/j/jit.asp. Accessed 3 July 2022.

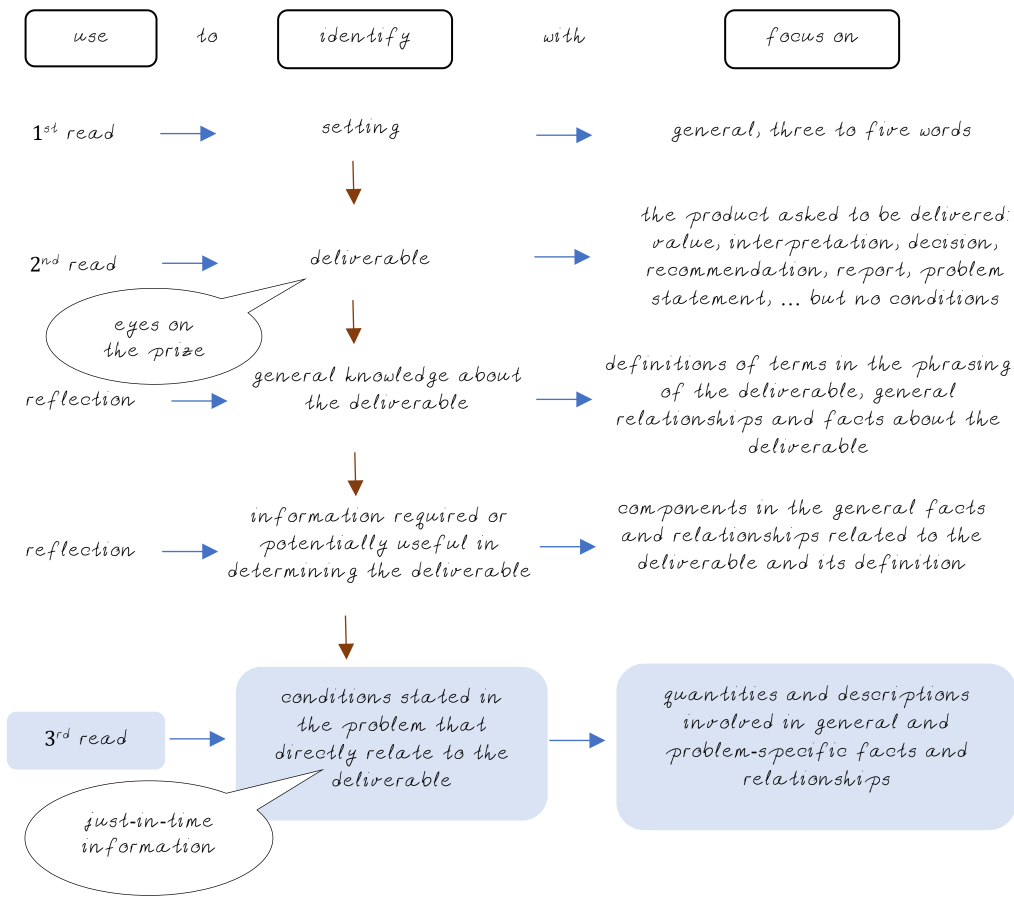



*Figure 5*

This is where the problem-solving process becomes complicated and definitely non-linear. In order to filter the conditions stated so that we can identify what is directly related to our deliverable, we have to keep in the foreground of our thinking space the list of items we identified would be useful in getting our deliverable. While working hard to not them escape like bubbles into the ether, we then have to cross-check each one of them with each of the conditions stated in the problem, categorizing each condition as either: “directly relevant” or “potentially relevant” to task at hand. Those in the first category we make note of, not just mentally but also writing them down. Those in the latter category we discard for the moment or store into a mental box called “not yet”.

This is hard. Super hard.

We will have to identify the conditions that explicitly describe the needed components of our deliverable and those that describe them implicitly. The conditions describing the deliverable implicitly we will have to rephrase so that the needed components can be expressed explicitly and be fitted into the puzzle giving us our deliverable. If this is not possible, we will have to consider methods that will allow us to get to the deliverable in indirect ways.

If we consider the earlier example of property taxes and the new mill rate, we can see that a number of conditions are stated in the problem, but only some, or perhaps none, are directly related to the deliverable:

> *A school board is determining next year's operating budget and calculates that it needs an additional $5 million to be collected through property taxes. Properties in its municipality have an assessed value of $8.455 billion. The current mill rate for the school board is set at 6.1998. If the assessed property values are forecasted to rise by 3% next year, what mill rate should the school set?*[6]

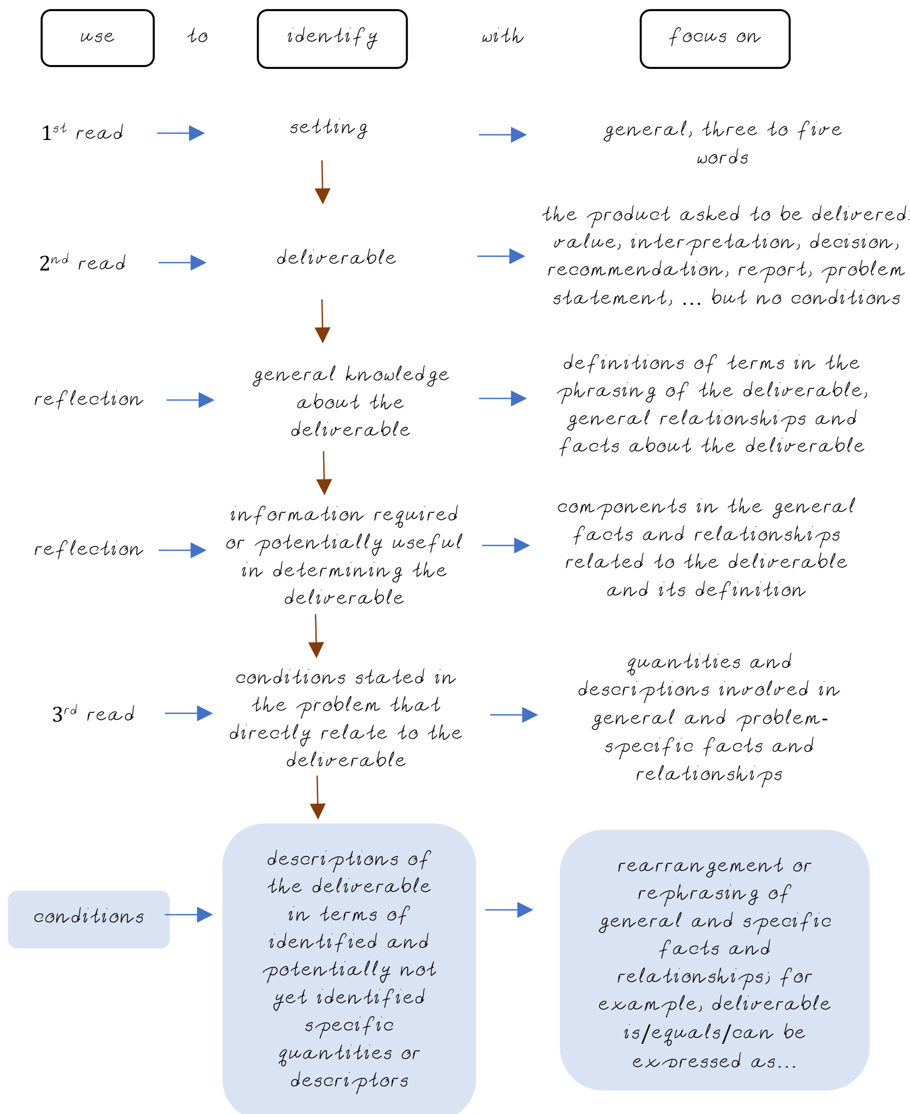
By scanning through the problem statement, we can see that there are no conditions that directly relate to the deliverable, i.e., there are no specific conditions that include in their descriptions the words representing the new mill rate. (The first sentence sets a condition on the new property tax amount. The second sentence addresses the current property values. The third sentence sets a condition on a current mill rate, and the last sentence addresses the condition relating current and new property value assessments.) Thus, we are left only with our general description of the deliverable in terms of the new tax amount and the new property value assessment, and with the problem of identifying these values in an indirect way.

In general, by focusing our first effort into filling in the puzzle using the conditions directly related to the deliverable is akin to starting by picking the low-hanging fruit. If there is fruit at the higher branches that needs to be picked, at least that low-hanging one we picked will not be in the way as we reach for the hard-to-reach ones and at least we have some sense of accomplishment by looking at the basket already partially full.

In this process of utilizing the just-in-time-information approach, we rework our general statements that describe our deliverable. Our goal is to express our deliverable in a way that clearly identifies the quantities and descriptors we have picked out of the conditions given, and those that remain to be found. We want to ask ourselves and be able to answer the following question:

***(5) Which conditions directly relate to my deliverable and allow me to fill in some of the pieces?***

[6] Adapted from J. Olivier (2018). *Business Math: A Step-by-Step Handbook*. Creative Commons License (CC BY-NC-SA)



use
to
identify
with
focus on
1st read
setting
general, three to five words
2nd read
deliverable
the product asked to be delivered: value, interpretation, decision, recommendation, report, problem statement, … but no conditions
reflection
general knowledge about the deliverable
definitions of terms in the phrasing of the deliverable, general relationships and facts about the deliverable
reflection
information required or potentially useful in determining the deliverable
components in the general facts and relationships related to the deliverable and its definition
3rd read
conditions stated in the problem that directly relate to the deliverable
quantities and descriptions involved in general and problem-specific facts and relationships
conditions
descriptions of the deliverable in terms of identified and potentially not yet identified specific quantities or descriptors
rearrangement or rephrasing of general and specific facts and relationships; for example, deliverable is/equals/can be expressed as…


*Figure 6*

### Taking a side trip, but not to just smell the roses

As we identify the pieces that we can fit into the relationships describing our deliverable, we will, more often than not, find that some pieces are still missing. At that point, those missing pieces become our new deliverables, or sub-deliverables, and the process of getting to them mimics the process so far described in getting our primary deliverable:

- Identify the missing pieces, the sub-deliverables;
- For each one,
    - Reflect on what you know about it, the definitions and relationships that include it within the general context of the problem;
    - Identify which pieces are needed to determine the sub-deliverable by rephrasing definitions and rearranging relationships as needed to explicitly describe it;
    - Identify the conditions stated in the problem that directly relate to the sub-deliverable and restate them, as needed, in a way that allows you to describe the sub-deliverable explicitly using the quantities and descriptors given and potentially others that create yet another level of sub-deliverables;
    - Once able to describe the sub-deliverable using only the given quantities and descriptors, use the description to fill in the missing piece in the main deliverable.

For example, in the problem with the new mill rate as the deliverable, we had determined the way to describe our deliverable in terms of the new tax amount and the new property value assessment. So, each becomes a sub-deliverable. We now proceed with repeating the reflection, recall and just-in-time-information process, but this time focused on each of the sub-deliverables. Recall the problem statement:

> *A school board is determining next year's operating budget and calculates that it needs an additional $5 million to be collected through property taxes. Properties in its municipality have an assessed value of $8.455 billion. The current mill rate for the school board is set at 6.1998. If the assessed property values are forecasted to rise by 3% next year, what mill rate should the school set?*[7]

Specifically, we note that the first sentence in the problem description implicitly describes the new tax amount by setting the condition that the new tax amount must be equal to the projected new budget and so the new mill rate can be described as the new budget. By rephrasing the first sentence we can say that the new amount of property tax is $5,000,000 more than the current amount of property tax. We can then take this statement and build it into our description of the deliverable that used the new tax amount, now requiring the value of the current property tax amount. Upon reviewing the problem statement again, we see that we are given the value of the current property tax amount, which then allows us to find the value of the first sub-deliverable, the new tax amount. Moving on to the second sub-deliverable, we note that the second sentence speaks to the current property tax amount, but not in a direct relation to the new property value assessment. Note, too, that the only reason why we know the reference is to *current* property value assessment because of the use of present verb tense - *have*. Neither is the current mill rate, specified in the third sentence,

[7] Adapted from J. Olivier (2018). *Business Math: A Step-by-Step Handbook*. Creative Commons License (CC BY-NC-SA)

directly related to new property value assessment. However, the last sentence contains a reference to new property value assessments. Rephrasing the stated condition, we have that the new property value assessment is 3% higher than the current property value assessment. Now, by rescanning the problem we grab onto the condition stated in the second sentence to describe, and determine the new property assessment value.

Below is a diagram that describes the “side-work” in pursuit of sub-deliverables. The ultimate goal is to ask, and be able to answer, the following question:

***(6) Which conditions help me determine the missing pieces and in what way?***

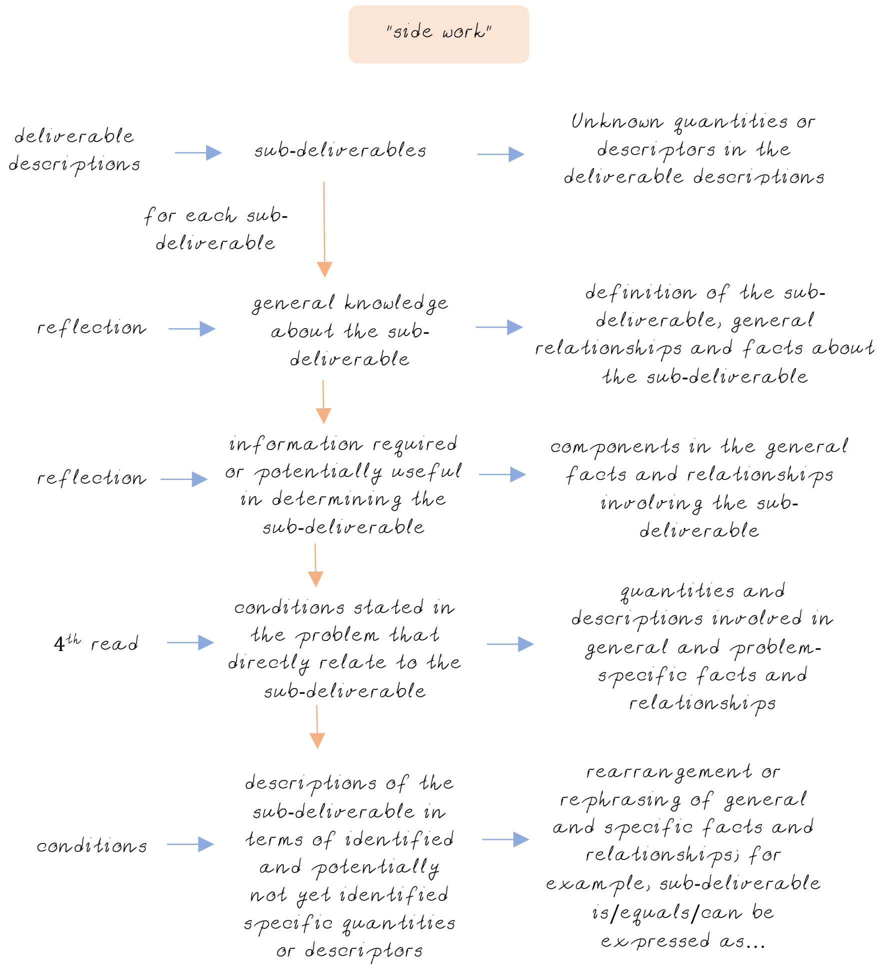



*Figure 7*

## Wrapping it up

### Putting it all together

Once we have identified all of the pieces required to assemble our deliverable, it is time to put them together. In this we use the description of our deliverable we identified at the start, connecting all of the necessary pieces through operations that bring them together into the whole that is our deliverable.

In our earlier example of the new mill rate, this translates to bringing in the values, or the process of calculating the values, of the new tax amount and the new property value assessment into the description of the deliverable.

At this stage, in a general problem-solving process, we should ask ourselves and be able to affirmatively and specifically answer the following question:

***(7) Am I able to, and how can I, describe the deliverable explicitly only using the conditions stated in the problem and those inherent to the deliverable in general?***

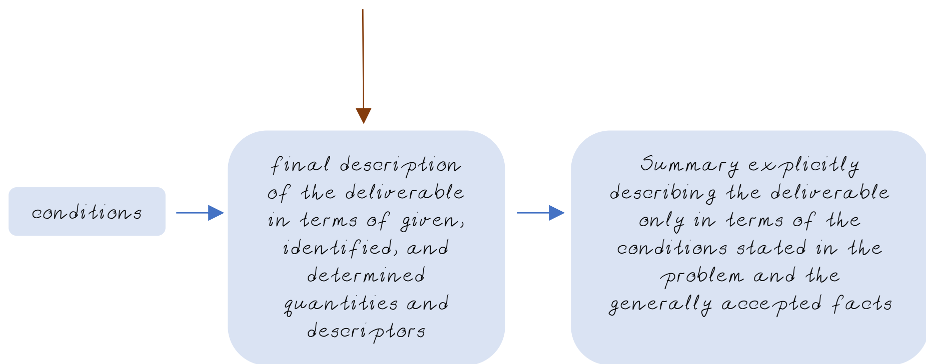



*Figure 8*

## Packaging the product

Finally, we review our problem description to confirm that we have indeed delivered what we were asked to deliver and, if needed, we format our deliverable as per requirements specified by the problem statement. Specifically, in presenting our deliverable we should use the language of the client, or whoever posed the problem, or the language of conventions within the context of the problem.

And so we come to the pinnacle question, answer to which will be the answer to the problem we were tasked with:

***(8) How do I present the deliverable within the specific and general constraints given by the overall setting of this problem?***

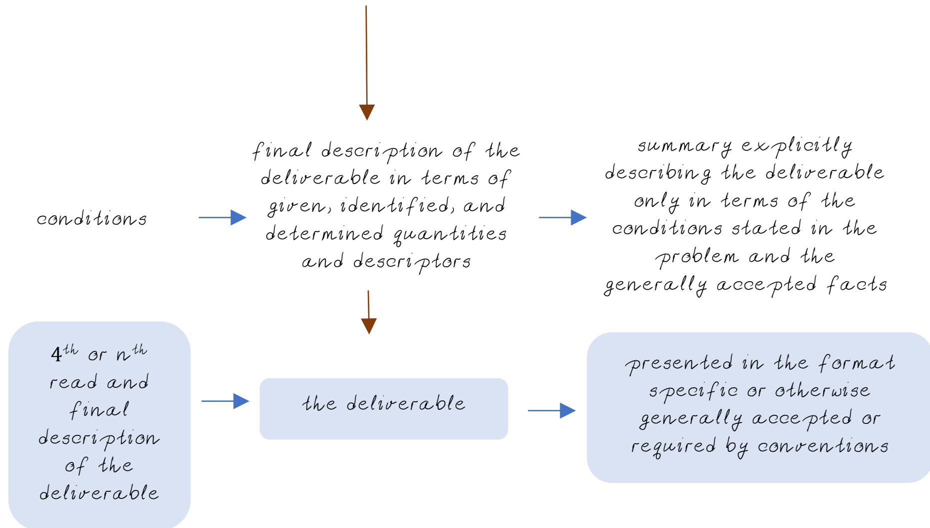



*Figure 9*

This concludes our travels through the problem-solving process aimed at achieving efficiency but, most of all, answering the questions "How do I know where to start?" and "How do I know where I should go next?". A visual of the totality of this process can be reviewed in Appendix A: Problem-solving Framework Diagram.

## Considerations and applications

### Some food for strategic thought

In our final stage of describing the solution to the problem, ideally our description presents the pieces and the connectors between them in as general form as possible, without or with only minimal use of specifics (as in numbers, names etc.). This will allow us to more easily review our result and check its validity before the final step of presenting our deliverable. In addition, it will provide a roadmap for solutions to problems of similar nature, where the context is the same but some specific values differ. This approach is particularly valuable when solving problems that, by their nature, will require a formulaic representation, such as in calculations using spreadsheets.

In problems of a mathematical nature, for example, where the deliverable is some value, this means that, before we use the numerical values provided in the problem, we should first convey the description of our deliverable in terms of general formulae and descriptors. As much as possible, specific quantities in these types of problems should be substituted into calculations only once the description of the deliverable using the given types of quantities is built.  This will also ensure that no unnecessary intermediate calculations are performed and thus will minimize the errors in the final result arising from rounding the mid-calculation results.

Below is a problem with two solution presentations exemplifying the value of leaving the specifics to the absolute end.

## Problem: Setting the tax rate to meet the new budgetary requirements

A school board is determining next year's operating budget and calculates that it needs an additional \$5 million to be collected through property taxes. Properties in its municipality have an assessed value of \$8.455 billion. The current mill rate for the school board is set at 6.1998. If the assessed property values are forecasted to rise by 3% next year, what mill rate should the school set?[8]

## Solution A (including intermediate calculations)

$$\text{new mill rate} = ?$$

What do we know about the new mill rate? We know that, in general, the property tax is calculated using mill rate by

$$\text{property tax amount} = \left(\frac{\text{mill rate}}{1000}\right)(\text{property value assessment})$$

Thus, by rearranging the equation to solve for the mill rate, we have

$$\text{mill rate} = \frac{\text{property tax amount}}{\text{property value assessment}} \cdot 1000$$

Since the new budget is funded by the property tax with the new mill rate applied to the totality of property value assessments, the new mill rate can therefore be calculated by

$$\Rightarrow \text{new mill rate} = \frac{\text{new budget}}{\text{new property value assessment}} \cdot 1000$$

So to find the new mill rate, we have to find out the value of the new budget and the new property value assessment. What do we know about the new budget?

Condition(s): School board needs \$5,000,000 more next year. So,

$$\text{new budget} = \text{current budget} + 5{,}000{,}000$$

So we need to know the current budget. What do we know about the current budget? We know the current mill rate, the current property value assessment, and that we can calculate the current budget, funded by current property tax, by

$$\text{current budget} = \left(\frac{\text{current mill rate}}{1000}\right)(\text{current property value assessment}) = \frac{6.1998}{1000} \cdot 8{,}455{,}000{,}000 = \$52{,}419{,}309$$

Hence

$$\Rightarrow \text{new budget} = \text{current budget} + 5{,}000{,}000 = 52{,}419{,}309 + 5{,}000{,}000 = \$57{,}419{,}309$$

To calculate the new mill rate, we also need the new property value assessment. We know that the property values are forecasted to rise by 3% next year. So,

$$\text{new property value assessment} = (\text{current property value assessment})(1 + 0.03) = 8{,}455{,}000{,}000 \cdot 1.03 = \$8{,}708{,}650{,}000$$

Therefore we have that

$$\Rightarrow \text{new mill rate} = \frac{\text{new budget}}{\text{new property value assessment}} \cdot 1000 = \frac{57{,}419{,}309}{8{,}708{,}650{,}000} \cdot 1000 = 6.59336510251 \approx 6.5934$$

[8] Adapted from J. Olivier (2018). *Business Math: A Step-by-Step Handbook*. Creative Commons License (CC BY-NC-SA)

## Solution B (no intermediate calculations)

$$new\ mill\ rate\ = ?$$

What do we know about the new mill rate? We know that, in general, the property tax is calculated using mill rate by

$$property\ tax\ amount\ = \left(\frac{mill\ rate}{1000}\right)(property\ value\ assessment)$$

Thus, by rearranging the equation to solve for the mill rate, we have

$$mill\ rate\ = \frac{property\ tax\ amount}{property\ value\ assessment} \cdot 1000$$

Since the new budget is funded by the property tax with the new mill rate applied to the totality of property value assessments, the new mill rate can therefore be calculated by

$$\Rightarrow new\ mill\ rate\ = \frac{new\ budget}{new\ property\ value\ assessment} \cdot 1000$$

So to find the new mill rate, we have to find out the value of the new budget and the new property value assessment. What do we know about the new budget?

Condition(s): School board needs $5,000,000 more next year. So,

$$new\ budget\ = current\ budget\ + budget\ increase$$

So we need to know the current budget. What do we know about the current budget? We know the current mill rate, the current property value assessment, and that we can calculate the current budget, funded by current property tax, by

$$current\ budget\ = \left(\frac{current\ mill\ rate}{1000}\right)(current\ property\ value\ assessment)$$

Hence

$$\Rightarrow new\ budget\ = \left(\frac{current\ mill\ rate}{1000}\right)(current\ property\ value\ assessment)\ +\ budget\ increase$$

To calculate the new mill rate, we also need the new property value assessment. We know that the property values are forecasted to rise by 3% next year. So,

$$new\ property\ value\ assessment = (current\ property\ value\ assessment)(1 + rate\ of\ change\ in\ property\ value\ assessment)$$

Therefore we have that

$$\Rightarrow new\ mill\ rate\ = \frac{\left(\frac{current\ mill\ rate}{1000}\right)(current\ property\ value\ assessment)\ +\ budget\ increase}{(current\ property\ value\ assessment)(1 + rate\ of\ change\ in\ property\ value\ assessment)} \cdot 1000$$

And so,

$$new\ mill\ rate\ = \frac{\left(\frac{6.1998}{1000}\right)(8{,}455{,}000{,}000)\ + 5{,}000{,}000}{(8{,}455{,}000{,}000)(1 + 0.03)} \cdot 1000 = 6.59336510251 \approx 6.5934$$

As can be seen from above, though the second solution is perhaps more verbose, it provides a clearer narrative of the solution and thus makes it easier to review and check for correctness and completeness. In addition, the final description of the deliverable, i.e., the expression for the new mill rate, is more valuable because it can be used and re-used when same types of quantities are involved but potentially through different values, and not just for the specific ones presented in this problem.

In general, for the development of problem-solving skills it is beneficial to help our students learn to develop solutions to given problems in the most general way and to include specifics during the solution process only when necessary or when they enhance the readability of the solution to a non-expert reader. Granted, this is harder and requires more discipline of the mind than the straight-forward “plug and play” approach. However, our guidance and target-setting in that regard will benefit our students in relation to development of their problem-solving skills. If they are able to determine solutions in more general form, they will be able to determine solutions in specific form. The converse does not hold true, however, and so a challenge is what we as teachers should be aiming for.

### How to help students develop and strengthen the discipline of the mind

So, if we can agree on the problem-solving framework described in this paper, in the words of my colleague William Thurber, how do we go about helping students develop the focus required? How do we help them resist the temptation to get into the weeds before, and only if, necessary? How do we guide them in exercising the discipline of the mind involved in first the identification of the target, and then the continual focus on the target while strategically considering the conditions imposed and using them to create the roadmap to the answer, our deliverable?

In my own practice I have been using three different approaches to teaching problem-solving using the discussed framework. They are based on:

- Demonstrating,
- Guiding, and
- Critically engaging

Additionally, I have been applying techniques that specifically focus on the development of resistance to mud- and weed-wading and on zeroing in on the target.

#### Demonstrating

In terms of demonstrating this framework in practice, I am driven by the Latin adage *repetitio est mater studiorum*, or that repetition is the mother of learning. I try to use consistent terminology in a repetitive fashion so that the fluency of process through familiarity of words is created. I demonstrate the process of rephrasing statements into equivalent forms that use familiar terminology, which in turn triggers recall of familiar or intuitive processes.

Additionally, when discussing a problem with my students, I force myself back into the state I was in when I first looked at the problem and, instead of presenting the solution that I had arrived to, I walk through the process that led me to the solution. In this I follow the question-answer dance that follows the stages outlined in the framework. I speak all of my thoughts out loud and demonstrate how I make note of the thoughts, whether it is by noting down the singular thoughts or noting down conclusions of my thought processes. Specifically, I demonstrate:

- how I review the problem statement the first time, whether by reading, listening or viewing, to arrive to a statement that describes the general setting;
- how I scan through the problem the second time, now with the objective of determining the deliverable, and then how I note down the deliverable(s) in abbreviated but clear form;
- how I consider the definitions related to each deliverable and my general knowledge of relationships directly involving the deliverable;

- how I note down the general conditions arising from definitions and the relationships, and how I rearrange them in words and in symbols to create descriptions of the deliverable that could be useful in creating a roadmap to a solution;
- how I review the problem statement again, looking for specific conditions directly related to the deliverable in the context of definitions and general relationships, and how I work with the specific conditions to fill in at least some of the pieces that make up my deliverable;
- how I work on identifying the pieces still missing, and how I repeat the process as for the main deliverable for each one of those pieces, noting down visually those missing pieces and the roadmap I am creating for each;
- how I continue to fill in the pieces that create my deliverable, in a systematic way that can be clearly followed by me and by others, how this helps me keep track of where I am and where I still have to go, and how this helps me recognize more easily errors or omissions I may have made in the process;
- and finally, how I ultimately deliver the deliverable, following the format that is either prescribed in the problem statement or through general conventions.

I am finding I still have to work hard when demonstrating this process in practice. This is firstly because I have to unlearn some bad habits discussed in the introduction of this paper and that I developed through the education systems (and also by simply being human). Secondly, when demonstrating something, we continually have to exercise judgement on the level of detail to include in verbalizing or writing down our thought processes, which is mentally strenuous.

The latter also brings up a common and frequently asked question by students – how much detail should they include in the presentation of their process? This is a complex question and could be a topic of another discussion paper, but the general guidance I share with the students and that I employ myself is: *if in doubt, write it out*. In other words, if you find yourself asking the question on whether you should note down a specific point, then it is fairly safe to assume that the answer is *yes*. In other words, it is very likely that intuitively you sense that there may be someone reading your process that needs that gap filled or that you yourself need that gap filled in order to make sure that you are not making a non-valid presumption in your argument.

These are some of the approaches I currently use to show students the problem-solving process in practice. **A question for you, the reader: how do you demonstrate problem solving to your students?**

### Guiding

Much of our learning takes place through conversations, asking or being asked to provide more details or direction of thought. This is what I capitalize on as I try to support students in creating and supporting the argument that leads to a solution to a problem.

In support of the question-answer dance approach, I typically would guide with questions and prompts, as necessary, while students make their attempt at a particular problem:

- ***Ask yourself the question****: what is this problem about? Then take a minute or two to scan through the problem to get the feel for the setting and let us know how you would describe this setting in three to five words. No details, no numbers.*

- *Scan through the problem the second time and **ask yourself**: what are my deliverables? Let us know. **Ask yourself**: how would I note them down in an abbreviated, but clear form?*
- ***Ask yourself**: what do the words in my deliverable mean? What are the general conditions that define the deliverable? What do I know in general how one could obtain this deliverable? How would I note down these conditions and relationships that govern your deliverable and can I phrase them in ways that give me explicit descriptions of my deliverable, in terms of the necessary parts that make it a whole?*
- *Now that you have an understanding of which parts would be useful and how they could be put together, scan through the problem description again and let us know which specific conditions make some or all of those parts. **Ask yourself**: how can I fit them into the puzzle that makes my deliverable? What is a good way to note this down, to get a sense of what's taken care of and what is still missing?*
- *If there are still parts missing, ask yourself about what you know in general about those parts. **Ask yourself**: what are the conditions that define these missing parts and which general relationships are they part of? Can I rephrase them so that I can express the parts that are missing in terms of some other parts that are potentially given in the problem in form of some specific conditions?*
- *For each of the missing parts, scan the problem again and let us know which conditions directly relate to your missing part. **Ask yourself** if you need to and, if yes, how could you rephrase the condition in a way that is helpful in determining your missing part? How does the missing part help in filling in the puzzle that makes up your deliverable and how would you note that? What does your deliverable puzzle look like now? What is the next piece that is missing?*
- *Now that you have all of the necessary pieces and you are only using the general conditions on the deliverable and the specific conditions set in the problem, **ask yourself**: what can I say about the deliverable? Is there a particular format for my presentation of the deliverable, required by the problem or by general conventions? What is my final answer to the problem?*
- *Scan the problem one more time and let us know if you completed the task. **Ask yourself**: did I deliver the deliverable(s)? How do I know?*
- *Scan your solution notes – do they seem complete? **Ask yourself**: will the reader of my solution understand, be able to follow, and agree with my argument?*

It is crucial to teach students not to depend on questions and prompts from others and thus they should be only employed when needed. In addition, framing the questions in a way that builds the student's ability to pose and answer their own questions will empower them to be their own guides through their uncertainties.

On the latter, students in general tend to answer our questions and prompts in question-like form, where the inflection on their answer statements indicates uncertainty and/or entreat for confirmation. For example:

*Me: If the value of the derivative is negative, what can you say about the function?*

*Student: The function is decreasing?*

It is my practice, when I remember to practice the teaching discipline, to not simply give a confirmation. Instead I ask students to answer their own answer-questions. I tell them I heard a question in their answer and tell them that is good, to ask oneself a question, but then they have to be the ones who answer it. Additionally, to support them in gaining that confidence that is needed to answer questions without external verification, I often prompt them to follow their answer with the word "because" and ask them to complete the sentence, i.e., to follow their assertion with the reason for their assertion.

These are some of the strategies I am currently employing in my teaching practice in relation to guiding students. **I am curious - how do you guide your students in problem solving?**

### Critically engaging

Some demonstration and guidance aspects also make an appearance in the process of critiquing student work, whether as part of formative or summative assessments. What can we do, in the process of assessing student work, that will prompt the student to critically re-evaluate their own work. How can we engage them in re-evaluating their process towards the problem solution, and their argument of the validity of their process and their deliverable?

Probably the most common feedback words I use on student assessments are

> *How did you get here?*
>
> *Why?*
>
> *Justify*.

I think of the power of those words and the multiple purposes they serve.

First, they show that whatever seemed evident to the student is perhaps not evident to someone else. It prompts the thought: why *did* I do/claim that?

Second, they prompt reflection and awareness of process of thought, and the practice of formulating inferential statements, including identification of gaps and unfounded assumptions.

Asking the *why* question often exasperates students. A common statement I hear from students is that *they know why, just don't know how to explain it*. My usual response to that is that *if you can't say it, then you don't know it*. Pressing hard on the demand that they attempt to present an argument, and guiding them through continual rephrasing of their statements of argument until they are happy with them, helps the students pave those brain highways for future argument formulation. Of course, this takes time. I wish we had endless time. As a result, how far I push and how much I guide through critique is limited by circumstances of time and environment.

**A question for you: how do you employ critique to engage your students in reflecting on their own problem-solving process?**

### Practicing the discipline of the mind

One commonly brought up distinction is that of *mechanics* vs. *analytics*, often comparing the first disparagingly to the second. In this discussion I am among those who believe both are equally important and that you cannot have analytics without mechanics (though you can have mechanics without analytics). For example, you must be mechanical in your knowledge of addition, subtraction, multiplication and division tables of integers to be able to perform analytical processes involved in applying addition, subtraction, multiplication and division on fractions. You then have to build your skill with operations on fractions to the degree of it being mechanical before you can apply the analytical processes to problems such as those involving finding solutions to equations or working with proportions.

Along that line of thinking, the guiding questions (1) to (8) presented above in the course of discussion of the problem-solving framework, should become mechanical, so that the mind can focus on the analytics required to answer those questions.

What presents itself as a challenge when teaching problem solving through non-trivial, complex problems is again that cognitive capacity limitation, which has the mind forget past objectives that were met once faced with a new objective. To address this I have employed, in teaching selective concepts, a particular technique to train the focus on the objective at hand.

This technique consists of presenting students simultaneously with a number of problems that are significantly different in content but are similar in the learning objective they address. I then ask students progressively through the list of problems to read the problem and describe the general setting in three to five words. In the beginning they tend to focus in their descriptions on specifics given in the problem and/or the task, i.e. the deliverable. I proceed to correct and guide them on how to accomplish the objective of succinctly describing the setting in very general terms, without a reference to the task and the specifics. By repeating the process for each of the problems presented without being allowed to let the brain flow beyond meeting this particular objective, the students begin to build the ability to discipline their thinking process towards the objective at hand.

After we have been able to surmise the general setting for each of the problems, we return to the first problem and work on identifying, and noting down, the deliverable for each of the problems. Once that objective was met for each problem, we return to the first problem and proceed through the others with the objective to define and recall whatever conditions are inherent to the identified deliverable in each. Once that is accomplished, we return to the first problem and continue on through the rest of them, with each identifying which specific conditions in the problem statement directly relate to the deliverable, noting them down and rephrasing as necessary, and setting aside those that are not directly related. You get the idea.

This seems to help. It would be a curious research question if indeed it does. Anecdotal evidence suggests the affirmative. Here are some students' answers to the question asked at the end of a first-year math course for business students: *Looking back at what you know now and are able to do compared to what you knew and could do before this course, what are you most proud about achieving?*

> *I am most proud of my analysis ability. Before this course, most of the math I learned was plug and play. Being able to analyze the question and formulate an equation(s) was something that I struggled with.*
>
> *I am most proud about learning how to properly analyze what words mean in word problems instead of focusing on numbers first.*
>
> *I am most proud about being able to think through questions more in-depth and be able to understand them better without going straight to numbers. I am proud*

*that I am able to understand and complete word problems easier than before this course.*

*I am better at math! I can rearrange equations properly, and have a system to tackle word problems. They don't frustrate me as much now.*

*I'm proud that now I can read application questions and solve them by formulating a formula through recognizing the task and walking step by step to complete that task.*

### Same ideas, just different lingo?

In conversing with colleagues, friends and family about the problem of problem-solving and how to teach it, it became apparent that the afore-described problem-solving framework is mirrored in many ways by processes in different spheres. Though the language used in different disciplines to describe different steps in the problem-solving process may differ, it seems that there are significant parallels, if not outright same framework.

Some of these include the processes involved in project design, military planning, design thinking, and writing papers and reports. It would be interesting to explore the similarities, the differences, the overlaps, and the divergences in problem-solving processes across different disciplines and problem types.

Perhaps we are all doing the same thing. If so, and here just pondering, would it be beneficial, both for us and our students, to have a reference to a universal problem-solving framework utilizing a common language, easily translatable to the specific language of each discipline or problem type?

## Appendix: Problem-solving Framework Diagram