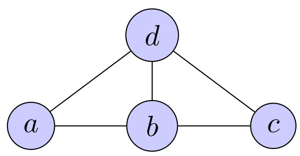
Walks and trails

Definitions

Term	Definition	Key restriction
Walk	A sequence of edges joining vertices	None
Trail	A walk with no repeated edges	Edges distinct
Path	A trail with no repeated vertices	Vertices distinct
Circuit	A trail that starts and ends at the same vertex	Closed trail
Cycle	A circuit with no repeated vertices (except start/end)	Closed path



$a \to b \to c$: Path
$a \to b \to d \to a$: Cycle
$a \to b \to c \to d \to b$: Trail (not a path)
$a \to b \to a \to d$: Walk (not a trail)

Note: Every path is a trail, and every trail is a walk. The reverse is not true.

Eulerian trails and circuits

An **Eulerian trail** visits every edge exactly once.
An **Eulerian circuit** is an Eulerian trail that starts and ends at the same vertex.

Type	Also called
Graph with an Eulerian circuit	Eulerian
Graph with an Eulerian trail (but no circuit)	Semi-Eulerian
Graph with neither	Neither

Conditions for Eulerian trails and circuits

For a **connected** graph:

Condition	Result
All vertices have even degree	Eulerian circuit exists
Exactly 2 vertices have odd degree	Eulerian trail exists (starts/ends at odd vertices)
More than 2 vertices have odd degree	Neither exists

Important: The graph must be connected! A disconnected graph with even degrees everywhere is not Eulerian.

Student Learning Support, Teaching and Learning Centre
studentlearning@ontariotechu.ca
ontariotechu.ca/studentlearning

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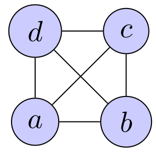
Examples

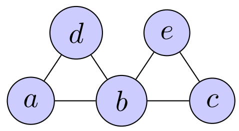
Example 1: Determine if the graph is Eulerian, semi-Eulerian, or neither.



Degrees: deg$(a) = 3$, deg$(b) = 3$
deg$(c) = 3$, deg$(d) = 3$

All four vertices have odd degree (more than 2), so **neither** an Eulerian trail nor circuit exists.

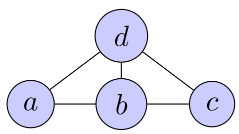
Example 2:



deg$(a) = 2$, deg$(b) = 4$, deg$(c) = 2$
deg$(d) = 2$, deg$(e) = 2$

All vertices have even degree, so an **Eulerian circuit** exists. The graph is Eulerian.
One possible circuit: $a \to b \to c \to e \to b \to d \to a$

Example 3:

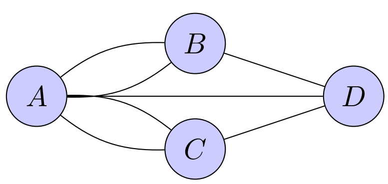


deg$(a) = 2$, deg$(b) = 3$
deg$(c) = 2$, deg$(d) = 3$

Exactly 2 vertices (b and d) have odd degree, so an **Eulerian trail** exists starting at b and ending at d (or vice versa). The graph is semi-Eulerian.
One possible trail: $b \to a \to d \to b \to c \to d$

The Seven Bridges of Königsberg

The classic problem that started graph theory! Can you walk through the city crossing each bridge exactly once?



deg$(A) = 5$ deg$(B) = 3$
deg$(C) = 3$ deg$(D) = 3$

All four vertices have odd degree $\Rightarrow$ No Eulerian trail or circuit exists. **It's impossible!**

Quick reference

Number of odd-degree vertices	Eulerian circuit?	Eulerian trail?
0	Yes	Yes (circuit is a trail)
2	No	Yes
> 2	No	No

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