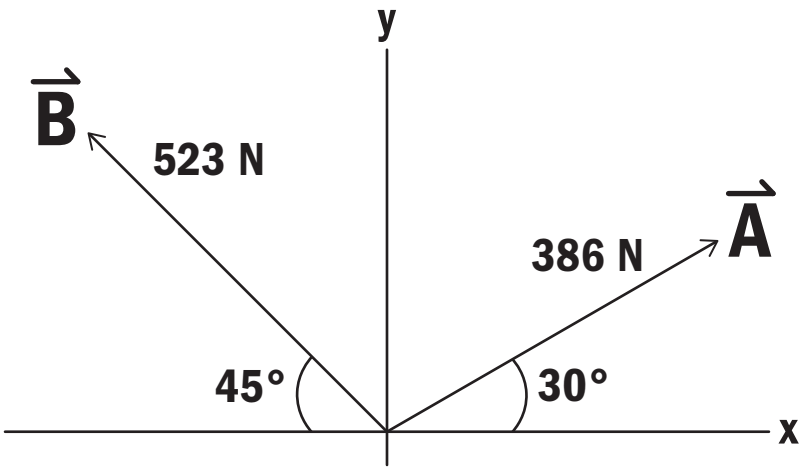
Vectors

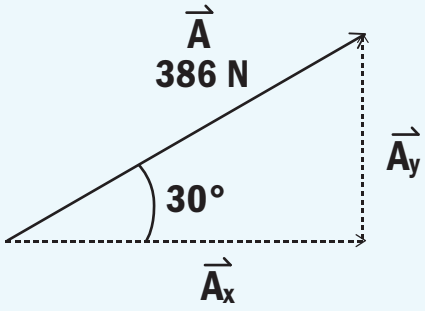
A vector is a quantity that has both magnitude and direction. For example, displacement, velocity, acceleration, and force are all vectors, as each of these quantities have a magnitude and direction associated with them.

To add vectors, they must first be decomposed into their x- and y- components. Like components are then added together. The resultant vector is then recomposed to determine its magnitude and direction.

Example: Two vectors, $\vec{A}$ and $\vec{B}$, are shown in the figure below. Find the resultant vector, $\vec{R} = \vec{A} + \vec{B}$, and include its direction as an angle from the positive x-axis.



STEP 1: DECOMPOSE THE VECTORS



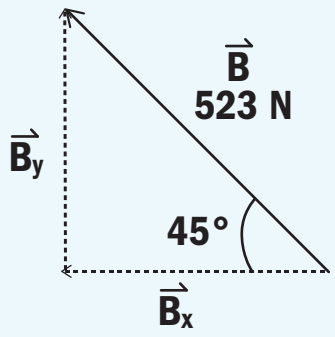
SOH CAH TOA can be used to determine $\vec{A}_x$ and $\vec{A}_y$. Since $\vec{A}_x$ is pointing right and $\vec{A}_y$ is pointing upward, both the x- and y-components are positive.

$$\cos(30°) = \frac{adj}{hyp} \qquad \sin(30°) = \frac{opp}{hyp}$$

$$\cos(30°) = \frac{A_x}{386} \qquad \sin(30°) = \frac{A_y}{386}$$

$$A_x = 386\cos(30°) \qquad A_y = 386\sin(30°)$$

$$A_x = 334 \text{ N} \qquad A_y = 193 \text{ N}$$



SOH CAH TOA can now be used to determine $\vec{B}_x$ and $\vec{B}_y$. Since $\vec{B}_x$ is pointing left and $\vec{B}_y$ is pointing upward, the x-component is negative and the y-component is positive.

$$B_x = -523\cos(45°) \qquad B_y = 523\sin(45°)$$

$$= -370 \text{ N} \qquad = 370 \text{ N}$$

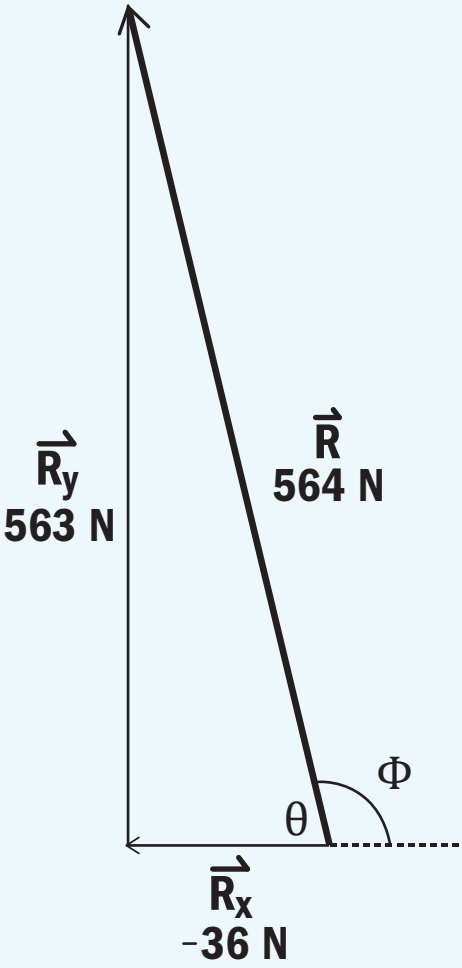
STEP 2: ADD LIKE COMPONENTS

$R_x = A_x + B_x$
$= 334 + (-370)$
$= -36\ N$

$R_y = A_y + B_y$
$= 193 + 370$
$= 563\ N$

STEP 3: RECOMPOSE THE VECTOR

To determine the resultant vector, $\vec{R}$, use vector addition to find $\vec{R}x + \vec{R}y$. Since these vectors are perpendicular, Pythagorean Theorem may be used to calculate the magnitude of $\vec{R}$. SOH CAH TOA can then be used to determine the direction of $\vec{R}$.



Magnitude of $\vec{R}$:

$$R = \sqrt{R_x^2 + R_y^2}$$
$$= \sqrt{(-36)^2 + (563)^2}$$
$$= 564\ N$$

Direction of $\vec{R}$:

$$\tan\theta = \frac{opp}{adj}$$
$$\tan\theta = \frac{R_y}{R_x}$$

$$\theta = \tan^{-1}\left|\frac{R_y}{R_x}\right|$$
$$= \tan^{-1}\left|\frac{563}{-36}\right|$$
$$= 86.3°$$

To determine the angle from the positive x-axis, Φ, subtract θ from 180°.

$$\Phi = 180° - 86.3°$$
$$\Phi = 93.7°$$

Therefore the resultant vector is:

$\vec{R} = 564\ N\ [93.7°\ CCW\ from\ the\ positive\ x\text{-}axis]$

Student Learning Centre

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