Sequences and Series

Sequence – a list of terms that are in order; e.g. {1, 3, 5, 7....} or $a_n = 1 + 2n$

Series – a sum of terms of an infinite sequence; e.g. 1+3+5+7+... or $\sum_{n=0}^{\infty} (1+2n)$

To determine if **series** converges, you need to know the following tests:

Test	When to Use	Conclusions
Geometric Series	When series has the form $\sum_{n=1}^{\infty} ar^{n-1}$ or $\sum_{n=0}^{\infty} ar^{n}$	Converges to $\frac{a}{1-r}$ if $\lvert r \rvert < 1$ diverges if $\lvert r \rvert \geq 1$
P-Series	When series has the form $\sum_{n=1}^{\infty} \frac{1}{n^p}$	Converges for $p > 1$; diverges for $p \leq 1$
Divergence Test	Any series for which $\lim_{n\to\infty} a_n \neq 0$	If $\lim_{n\to\infty} a_n \neq 0$ series diverges; inconclusive if $\lim_{n\to\infty} a_n = 0$
Ratio Test	Any series especially those with exponentials and/or factorials	Converges absolutely if $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert < 1$; diverges if $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert > 1$; inconclusive if $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = 1$
Root Test	For series with nth powers or greater	Converges absolutely if $\lim_{n\to\infty} \sqrt[n]{\lvert a_n \rvert} < 1$; diverges if $\lim_{n\to\infty} \sqrt[n]{\lvert a_n \rvert} > 1$; inconclusive if $\lim_{n\to\infty} \sqrt[n]{\lvert a_n \rvert} = 1$

To determine if a **sequence** converges or diverges, check the limit as $n \to \infty$.

- A sequence a_n converges to L if $\lim_{n\to\infty} a_n = L$.
- If limit does not exist, a sequence diverges.

Identify the advisable **convergence test** to use on the following **series** then use the test to determine whether the series converges or diverges.

1. $\sum_{k=0}^{\infty} \frac{2^k}{e^k}$ Test: _____ Conclusion: _____

2. $\sum_{n=1}^{\infty} n^2 e^{-n}$ Test: _____ Conclusion: _____

3. $\sum_{n=1}^{\infty} \frac{1}{n^{2/3}}$ Test: _____ Conclusion: _____

4. $\sum_{n=0}^{\infty} \left(\frac{-3n^3 + 5}{9n^3 + 4} \right)^n$ Test: _____ Conclusion: _____

ANSWERS:

1. Geometric series, converges
2. Ratio test, converges absolutely
3. P-series, diverges
4. Root test, converges absolutely

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