# Rationalizing Expressions

## What does rationalizing mean?

Rationalizing means rewriting an expression so that radicals are removed from either the numerator or the denominator.
This is done by multiplying by a carefully chosen form of $1$.
For example:

$$\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

## Why do we rationalize?

Rationalizing can make expressions:

- easier to simplify,
- easier to compare,
- and easier to use in later algebra or calculus.

## Rationalizing a single radical

If the numerator or denominator contains one radical term, multiply the top and bottom by that radical.
Example: Rationalize the denominator:

$$\frac{5}{\sqrt{7}}$$

$$\frac{5}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{5\sqrt{7}}{7}$$

Why does this work?

$$\sqrt{7} \cdot \sqrt{7} = 7$$

Example: Rationalize the numerator:

$$\frac{\sqrt{3}}{4}$$

$$\frac{\sqrt{3}}{4} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{3}{4\sqrt{3}}$$

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## Rationalizing with conjugates

If the numerator or denominator has two terms and one contains a radical, use the **conjugate**.

## What is a conjugate?

The conjugate changes the sign between two terms:

$$a+\sqrt{b} \qquad \longleftrightarrow \qquad a-\sqrt{b}$$

$$a-b\sqrt{c} \qquad \longleftrightarrow \qquad a+b\sqrt{c}$$

## Why do conjugates work?

Conjugates use the difference of squares identity:

$$(a+b)(a-b)=a^2-b^2$$

The middle terms cancel, which removes the radical.

Example: Rationalize the denominator:

$$\frac{2}{3+\sqrt{5}}$$

Use conjugate $3-\sqrt{5}$:

$$\frac{2}{3+\sqrt{5}}\cdot\frac{3-\sqrt{5}}{3-\sqrt{5}}=\frac{2(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})}$$

Simplify the denominator:

$$(3+\sqrt{5})(3-\sqrt{5})=3^2-(\sqrt{5})^2=9-5=4$$

So:

$$\frac{2(3-\sqrt{5})}{4}=\frac{3-\sqrt{5}}{2}$$

Example: Rationalize the numerator:

$$\frac{\sqrt{x+h}-\sqrt{x}}{h}$$

Use the conjugate $\sqrt{x+h}+\sqrt{x}$:

$$\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$$

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$$= \frac{(\sqrt{x+h})^2 - (\sqrt{x})^2}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})} = \frac{h}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

## Common mistakes

- Multiplying only the numerator or only the denominator.
- Forgetting to multiply every term.
- Using the wrong conjugate.
- Forgetting to simplify after rationalizing.
- Forgetting that $(\sqrt{a})^2 = a$.

## Practice problems

Rationalize and simplify fully.

**1.** $\frac{1}{\sqrt{2}}$ **2.** $\frac{4}{\sqrt{3}}$ **3.** $\frac{5}{2\sqrt{7}}$ **4.** $\frac{\sqrt{5}}{3}$

**5.** $\frac{2}{1+\sqrt{5}}$ **6.** $\frac{1}{3-\sqrt{2}}$ **7.** $\frac{4}{2+\sqrt{3}}$ **8.** $\frac{\sqrt{x+h}-\sqrt{x}}{h}$

**Answers:**
1. $\frac{\sqrt{2}}{2}$ 2. $\frac{4\sqrt{3}}{3}$ 3. $\frac{5\sqrt{7}}{14}$ 4. $\frac{5}{3\sqrt{5}}$ 5. $\frac{\sqrt{5}-1}{2}$ 6. $\frac{3+\sqrt{2}}{7}$ 7. $8-4\sqrt{3}$ 8. $\frac{1}{\sqrt{x+h}+\sqrt{x}}$

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