Number systems

Common bases

System	Base	Digits used
Binary	2	0, 1
Octal	8	0, 1, 2, 3, 4, 5, 6, 7
Decimal	10	0, 1, 2, 3, 4, 5, 6, 7, 8, 9
Hexadecimal	16	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

Notation: $(n)_b$ means n is written in base b. For example, $(1010)_2$ is binary.

Place values

In base b, the place values are powers of b:

$$\ldots, b^3, b^2, b^1, b^0 \underbrace{.}_{\text{point}} b^{-1}, b^{-2}, \ldots$$

Base 10	10^3	10^2	10^1	10^0	.	10^{-1}	10^{-2}
	1000	100	10	1		0.1	0.01
Base 2	2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}
	8	4	2	1		0.5	0.25
Base 16	16^3	16^2	16^1	16^0	.	16^{-1}	16^{-2}
	4096	256	16	1		0.0625	0.00391

Converting to decimal (any base $\rightarrow$ base 10)

Multiply each digit by its place value and add.

Example: Convert $(1011)_2$ to decimal.

$$1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 = 8 + 0 + 2 + 1 = 11$$

Example: Convert $(752)_8$ to decimal.

$$7 \cdot 8^2 + 5 \cdot 8^1 + 2 \cdot 8^0 = 448 + 40 + 2 = 490$$

Example: Convert $(2A3)_{16}$ to decimal.

$$2 \cdot 16^2 + 10 \cdot 16^1 + 3 \cdot 16^0 = 512 + 160 + 3 = 675$$

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Reminder: In hex, A = 10, B = 11, C = 12, D = 13, E = 14, F = 15.

Converting from decimal (base 10 $\rightarrow$ any base)

Repeated division method: Divide by the base repeatedly; the remainders (read bottom to top) form the answer.

Example: Convert 47 to binary.

Division	Quotient	Remainder
$47 \div 2$	23	1
$23 \div 2$	11	1
$11 \div 2$	5	1
$5 \div 2$	2	1
$2 \div 2$	1	0
$1 \div 2$	0	1

Read remainders from bottom to top: $(47)_{10} = (101111)_2$

Example: Convert 490 to octal.

Division	Quotient	Remainder
$490 \div 8$	61	2
$61 \div 8$	7	5
$7 \div 8$	0	7

Read remainders from bottom to top: $(490)_{10} = (752)_8$

Example: Convert 675 to hexadecimal.

Division	Quotient	Remainder
$675 \div 16$	42	3
$42 \div 16$	2	10 (= A)
$2 \div 16$	0	2

Read remainders from bottom to top: $(675)_{10} = (2\text{A}3)_{16}$

Binary $\leftrightarrow$ Octal

Since $8 = 2^3$, group binary digits in threes (from the right).

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Binary	Octal
000	0
001	1
010	2
011	3
100	4
101	5
110	6
111	7

Example: Convert $(101111)_2$ to octal.
Group in threes from the right: $101\ 111$
Convert each group: $101 = 5$, $111 = 7$
Therefore, $(101111)_2 = (57)_8$
Example: Convert $(752)_8$ to binary.
Convert each digit: $7 = 111$, $5 = 101$, $2 = 010$
Therefore, $(752)_8 = (111101010)_2$

Binary $\leftrightarrow$ Hexadecimal

Since $16 = 2^4$, group binary digits in fours (from the right).

Binary	Hex	Binary	Hex
0000	0	1000	8
0001	1	1001	9
0010	2	1010	A
0011	3	1011	B
0100	4	1100	C
0101	5	1101	D
0110	6	1110	E
0111	7	1111	F

Example: Convert $(10101100)_2$ to hexadecimal.
Group in fours from the right: $1010\ 1100$
Convert each group: $1010 = \mathrm{A}$, $1100 = \mathrm{C}$
Therefore, $(10101100)_2 = (\mathrm{AC})_{16}$
Example: Convert $(\mathrm{F3})_{16}$ to binary.
Convert each digit: $\mathrm{F} = 1111$, $3 = 0011$
Therefore, $(\mathrm{F3})_{16} = (11110011)_2$

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Octal $\leftrightarrow$ Hexadecimal

Convert through binary (or decimal) as an intermediate step.

Example: Convert $(752)_8$ to hexadecimal.

$(752)_8 \to (111101010)_2 \to (0001\ 1110\ 1010)_2 \to (1\text{EA})_{16}$

Quick reference

Conversion	Method
Base b to decimal	Multiply digits by powers of b, add
Decimal to base b	Repeated division by b, read remainders up
Binary to octal	Group by 3 bits
Octal to binary	Expand each digit to 3 bits
Binary to hex	Group by 4 bits
Hex to binary	Expand each digit to 4 bits
Octal to hex	Go through binary

Practice problems. Perform the indicated conversions.

1. $(110101)_2$ to decimal

2. $(1001011)_2$ to decimal

3. $(347)_8$ to decimal

4. $(\text{B2F})_{16}$ to decimal

5. $(83)_{10}$ to binary

6. $(200)_{10}$ to binary

7. $(156)_{10}$ to octal

8. $(500)_{10}$ to hex

9. $(11010110)_2$ to octal

10. $(634)_8$ to binary

11. $(10111100)_2$ to hex

12. $(\text{A5})_{16}$ to binary

Answers: 1. 53 2. 75 3. 231 4. 2863 5. $(1010011)_2$ 6. $(11001000)_2$ 7. $(234)_8$ 8. $(\text{1F4})_{16}$ 9. $(326)_8$ 10. $(110011100)_2$ 11. $(\text{BC})_{16}$ 12. $(10100101)_2$

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