Logic Operators

If p and q are propositions. A proposition is a declarative statement that states a fact that is either true or false.

EXAMPLES	NON-EXAMPLES
Toronto is the capital of Ontario. (*True Proposition*)	Read this tip sheet. (*Not a Proposition*)
1 + 2 = 5 (*False Proposition*)	$x + 3 = 10$

If x = # then it will be a true/false proposition.
If x = 7, then it will be a true proposition.
If x = 9 then it will be a false proposition.

Precedence

PRECEDENCE	OPERATOR
1	¬ (negation)
2	∧ (and)
3	v (or)
4	→ (conditional)
5	↔ (bi-conditional)

We usually use parenteses first, then use these rules of precedence to specify the order you apply the logical operators.

For Example: p v q ∧ r *means* p v (q ∧ r)
rather than (p v q) ∧ r

Truth tables

In the truth tables below, F (or zero) denotes a false proposition while T (or one) denotes a true proposition.

p	q	NEGATION $\bar{p}$, (¬ p) not p	CONJUNCTION (p ∧ q) p and q	DISJUNCTION (p v q) p or q	CONDITIONAL (p → q) If p, then q; p implies q; q whenever p;	BI-CONDITIONAL (p ↔ q) p if and only if q	EXCLUSIVE OR (p ⊕ q) p or q but not both
T	T	F	T	T	T	T	F
T	F	F	F	T	F	F	T
F	T	T	F	T	T	F	T
F	F	T	F	F	T	T	F

p	q	CONVERSE q → p	CONTRAPOSITIVE $\bar{q} \rightarrow \bar{p}$	INVERSE $\bar{p} \rightarrow \bar{q}$
		If q, then p	If not q, then not p	If not p, then not q
T	T	T	T	T
T	F	T	F	T
F	T	F	T	F
F	F	T	T	T

Examples

a) Construct a truth table for the compound proposition: $(p \rightarrow q) \leftrightarrow (\bar{q} \rightarrow \bar{p})$

p	q	$\bar{p}$	$\bar{q}$	$p \rightarrow q$	$\bar{q} \rightarrow \bar{p}$	$(p \rightarrow q) \leftrightarrow (\bar{q} \rightarrow \bar{p})$
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

b) Use a truth table to determine if these two compound propositions are equivalent: $(p \rightarrow r) \vee (q \rightarrow r)$ and $(p \wedge q) \rightarrow r$

p	q	r	$p \rightarrow r$	$q \rightarrow r$	$p \wedge q$	$(p \rightarrow r) \vee (q \rightarrow r)$	$(p \wedge q) \rightarrow r$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F
T	F	T	T	T	F	T	T
T	F	F	F	T	F	T	T
F	T	T	T	T	F	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	F	T	T
F	F	F	T	T	F	T	T

Therefore, both propositions are equivalent as the truth values for both propositions result the same.

c) Use the following propositions to complete the questions below.

p: The sun is shining.

q: You will go to the beach.

r: You wear sunglasses.

s: You wear sunscreen.

i) *Express the following compound propositions as symbols:*

1) If the sun is not shining then you do not go to the beach.
2) If you go to the beach then you will wear sunscreen or sunglasses.
3) If the sun is shining and you are going to the beach then you will wear sunglasses and sunscreen.

ii) *Express the following compound propositions as English sentences:*

4) $q \leftrightarrow p$
5) $((\bar{p} \wedge q) \rightarrow \bar{r})$
6) $p \rightarrow (s \vee r)$

Answers

i) Symbols

1. $\bar{p} \rightarrow \bar{q}$
2. $q \rightarrow (s \vee r)$
3. $(p \wedge q) \rightarrow (r \wedge s)$

ii) English

4. You will go to the beach if and only if the sun is shining.
5. If the sun is not shining and you go to the beach then you will not wear sunglasses.
6. If the sun is shining then you will wear sunscreen or sunglasses.

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