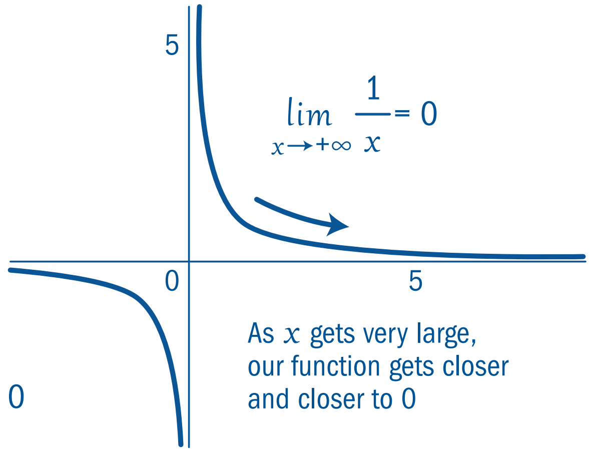
Introductory calculus

WHAT IS A LIMIT? $\lim\limits_{x \to c} f(x) = L$

This means that as x **approaches** some value, c, the function **approaches** L, our limit. Limits help us understand how the function behaves.



As x gets very large, our function gets closer and closer to 0

LIMIT LAWS

$$\lim_{x \to c} (f(x) \pm g(x)) = L \pm M \qquad \lim_{x \to c} (f(x)\, g(x)) = L\, M$$

$$\lim_{x \to c} (kf(x)) = k \lim_{x \to c} (f(x)) = kL \qquad \lim_{x \to c} \left(\frac{f(x)}{g(x)} \right) = \frac{L}{M} \text{ if } M \neq 0$$

EVALUATING LIMITS

1. **Use direct substitution:** $\lim\limits_{x \to c} f(x) = f(c)$
2. If this gives you the indeterminate form "$\frac{0}{0}$" try factoring or rationalizing to simplify the expression first.

   Example:

$$\lim_{x \to -3} \frac{x^2 - 9}{x + 3} = \lim_{x \to -3} \frac{(x - 3)(x + 3)}{x + 3} = \lim_{x \to -3} x - 3 = -6$$

EVALUATING LIMITS AT INFINITY

We can't reach infinity but we want to see where our function is going as x gets very large.

Examples: $\lim\limits_{x \to -\infty} 3x$ does not exist (D.N.E) since $3x$ will simply approach $-\infty$

For rational functions, $\frac{P(x)}{Q(x)}$:

$\lim\limits_{x \to \infty} \frac{x^2 + 3}{x^4 - x} = 0$ since the degree of P is less than Q

$\lim\limits_{x \to \infty} \frac{4x^3 + 5x}{7x^3 - 1} = \frac{4}{7}$ since the degree of P is equal to Q

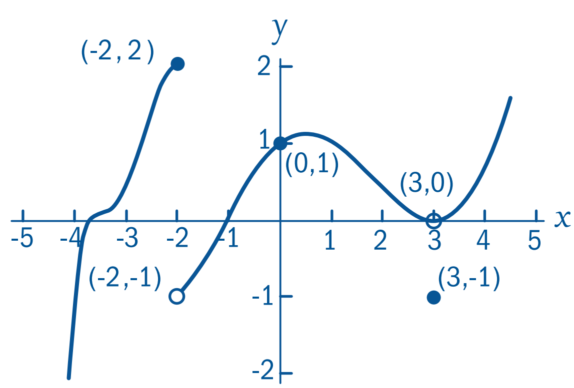
CONTINUITY

The function in the example is piecewise and clearly discontinuous. But to be **continuous at a specific point $x = c$** we need 3 conditions to be true:

1. $f(c)$ exists **2.** $\lim\limits_{x \to c} f(x)$ exist **3.** $\lim\limits_{x \to c} f(x) = f(c)$

On this graph, (0,1) is a continuous point, use the above condition to show why.

Example:



Does the limit exist?

The limit as we approach $x = -2$ from the left: $\lim\limits_{x \to -2^-} f(x) = 2$

The limit as we approach $x = -2$ from the right: $\lim\limits_{x \to -2^+} f(x) = -1$

$\lim\limits_{x \to -2^-} f(x) \neq \lim\limits_{x \to -2^+} f(x) \therefore \lim\limits_{x \to -2} f(x)$ D.N.E.

DIFFERENTIATION

The derivative of a function $f(x)$ gives the slope of the tangent (instantaneous rate of change).

Previously we always used two points to find slope but the tangent only touches one point on the graph. **Secant slope:** $\frac{f(x_1)-f(x_0)}{x_1-x_0}$	To approximate the slope of the tangent we can take two points very close to each other, say h away, and calculate the slope of this secant. **Approximate Tangent Slope:** $\frac{f(x_0+h)-f(x_0)}{x_0+h-x_0}$	Now that we know about limits we can find the value of the slope of the tangent if we make h approach 0

So a derivative is defined as $f'(x)=\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$.

DIFFERENTIATION RULES

Constant Rule: $f(x)=c$ then $f'(x)=\frac{df(x)}{dx}=0$	$f(x)=5$ $\quad f'(x)=0$
Power Rule: $\frac{d}{dx}x^n=nx^{n-1}$	$f(x)=x^3$ $\quad f'(x)=3x^2$
Constant Multiple Rule: $\frac{d}{dx}(cf(x))=cf'(x)$	$f(x)=3\sin x$ $\quad f'(x)=3\cos x$
Sum Rule: $\frac{d}{dx}[f(x)\pm g(x)]=f'(x)+g'(x)$	$f(x)=5\cos(x)-2x^2$ $f'(x)=-5\sin(x)-4x$
Product Rule: $\frac{d}{dx}[f(x)g(x)]=f'(x)g(x)+f(x)g'(x)$	$f(x)=(x^4+x)e^{2x}$ $f'(x)=(4x^3+1)e^{2x}+(x^4+x)(2e^{2x})$
Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{f'(x)g(x)-f(x)g'(x)}{[g(x)]^2}$	$f(x)=\frac{\sin(x)}{2e^x}$ $\quad f'(x)=\frac{\cos(x)2e^x-\sin(x)2e^x}{4e^{2x}}$
Chain Rule: $\frac{d}{dx}[f(g(x))]=f'(g(x))g'(x)$	$f(x)=\sqrt{3x^2+5}$ $\quad f'(x)=\frac{1}{2\sqrt{3x^2+5}}(6x)$

Useful Derivatives:

$\frac{d}{dx}(\sin x)=\cos x$

$\frac{d}{dx}(\cos x)=-\sin x$

$\frac{d}{dx}(\tan x)=\sec^2 x$

$\frac{d}{dx}(\sec x)=\sec x\tan x$

$\frac{d}{dx}(\cot x)=-\csc^2 x$

$\frac{d}{dx}(\csc x)=-\csc x\cot x$

$\frac{d}{dx}(\ln x)=\frac{1}{x}$

$\frac{d}{dx}(a^x)=(\ln a)a^x$

$\frac{d}{dx}(e^x)=e^x$

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