Intro to Sequences and Series

Sequences and series

A **sequence** is an ordered list of numbers.

$$2,\ 5,\ 8,\ 11, \ldots$$

The terms of a sequence are often written using notation such as $a_1,\ a_2,\ a_3,\ \ldots,\ a_n$. For example, the above sequence $2,\ 5,\ 8,\ 11, \ldots$ can be written as $\{a_n\} = \{3n-1\}$.

A **series** is the sum of the terms of a sequence.

$$2+5+8+11+\cdots$$

Using summation notation, the same series can be written as

$$\sum_{n=1}^{\infty}(3n-1).$$

The symbol Σ (sigma) means "add the terms."

$$\sum_{n=1}^{5} n = 1+2+3+4+5 = 15.$$

Key formulas

Type	Formula
Arithmetic sequence	$a_n = a_1 + (n-1)d$
Arithmetic series	$\sum_{n=1}^{N} a_n = S_N = \frac{N}{2}(a_1 + a_N)$
Geometric sequence	$a_n = a_1 r^{n-1}$
Finite geometric series	$\sum_{n=1}^{N} ar^{n-1} = S_N = a\left(\frac{1-r^N}{1-r}\right), \quad r \neq 1$
Infinite geometric series	$\sum_{n=1}^{\infty} ar^{n-1} = S = \frac{a}{1-r}, \quad \lvert r \rvert < 1$

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Arithmetic vs. geometric sequences

	Arithmetic	**Geometric**
Pattern	Add/subtract a constant	Multiply/divide by a constant
Example	$3, 7, 11, 15, \ldots$	$3, 6, 12, 24, \ldots$
Common value	Difference d	Ratio r

Examples

Example 1: Find the 15th term of

$$4,\ 7,\ 10,\ 13, \ldots$$

This is an arithmetic sequence with $a_1 = 4, \quad d = 3.$
Using

$$a_n = a_1 + (n-1)d,$$

$$a_{15} = 4 + (15-1)(3) = 46.$$

Example 2: Find the **sum** of the first 20 terms of

$$5,\ 8,\ 11,\ 14, \ldots$$

Since each term increases by 3, this is an arithmetic sequence with $a_1 = 5, \quad d = 3.$
To find the sum, we use the arithmetic series formula with $N = 20$.

$$S_N = \frac{N}{2}(a_1 + a_N),$$

$$a_{20} = 5 + 19(3) = 62.$$

$$S_{20} = \frac{20}{2}(5 + 62) = 670.$$

Therefore, the sum of the first 20 terms is 670.

Example 3: Find the 8th term of

$$2,\ 6,\ 18,\ 54, \ldots$$

Notice that each term is obtained by multiplying the previous term by 3. This is a geometric sequence with $a = 2, \quad r = 3.$
Using

$$a_n = ar^{n-1},$$

$$a_8 = 2(3)^7 = 4374.$$

Example 4: Find the sum of the infinite geometric series

$$12 + 6 + 3 + \frac{3}{2} + \cdots$$

Notice that each term is obtained by multiplying the previous term by $\frac{1}{2}$ (i.e. dividing by 2). Since the ratio between consecutive terms is constant, this is a geometric series with

$$a = 12, \qquad r = \frac{1}{2}.$$

Since $|r| < 1$, the series converges and we can find the sum:

$$\sum_{n=1}^{\infty} 12\left(\frac{1}{2}\right)^{n-1} = \frac{12}{1-\frac{1}{2}} = 24.$$

Therefore, the sum is 24.

Practice problems. Find the requested term or sum.

1. Find the 12th term of $5, 9, 13, \ldots$

2. Find the 25th term of $7, 10, 13, \ldots$

3. Find $\sum_{n=1}^{15}(3n-1)$

4. Find $\sum_{n=1}^{20}(4n+6)$

5. Find the 9th term of $3, 6, 12, \ldots$

6. Find the 8th term of $5, 15, 45, \ldots$

7. Find $\sum_{n=1}^{6} 2^n$

8. Find $\sum_{n=1}^{5} 3\left(\frac{1}{2}\right)^{n-1}$

9. Find $\sum_{n=1}^{\infty} 8\left(\frac{1}{4}\right)^{n-1}$

10. Does $\sum_{n=1}^{\infty} 5(2)^{n-1}$ converge?

Answers: 1. 49 2. 79 3. 345 4. 960 5. 768 6. $10\,935$ 7. 126 8. $\frac{93}{16}$ 9. $\frac{32}{3}$ 10. No

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