

Exponents

When an exponent is a positive integer, it indicates how many times the base is multiplied by itself.

$$\begin{array}{c}
 \text{Base} \quad \quad \quad \text{Exponent} \\
 \swarrow \quad \quad \quad \searrow \\
 x^n = \underbrace{x \cdot x \cdot x \cdots x}_n \\
 \text{\small } x \text{ multiplied by itself } n \text{ times}
 \end{array}$$

Example: $2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$

When using a calculator, the exponent key might be $^{\wedge}$ or x^y

Properties of Exponents:

Property	Example
Zero exponent $x^0=1$ if $x \neq 0$	$5^0=1$
Negative exponent $x^{-n} = \frac{1}{x^n}$	$2^3 = \frac{1}{2^3} = \frac{1}{8}$
Product of like bases $x^m x^n = x^{m+n}$	$a^3 a^4 = a^{3+4} = a^7 \leftarrow a^3 a^4 = (aaa)(aaaa) = a^7$
Quotient of like bases $\frac{x^m}{x^n} = x^{m-n}$	$\frac{y^{11}}{y^3} = y^{11-3} = y^8$
Power of a power $(x^m)^n = x^{mn}$	$(x^4)^5 = x^{4 \cdot 5} = x^{20}$
Power of a product $(xy)^n = x^n y^n$	$(st)^6 = s^6 t^6$
Quotient of like bases $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$	$\left(\frac{x}{y}\right)^7 = \frac{x^7}{y^7}$
Nth root $x^{1/n} = \sqrt[n]{x}$	$x^{1/2} = \sqrt{x} \quad x^{1/3} = \sqrt[3]{x}$
Fractional exponent $x^{m/n} = \sqrt[n]{x^m}$	$16^{3/4} = \sqrt[4]{16^3} = (\sqrt[4]{16})^3 = 2^3 = 8$
Adding like-terms $ax^n + bx^n = (a+b)x^n$	$3x^2 + 8x^2 = 11x^2$

Caution: $(x+y)^n \neq x^n + y^n$

Example: Simplify $\left(\frac{2x^{-6}x^6}{y^{-2}}\right)^{-3}$

$$\left(\frac{2x^{-6}x^6}{y^{-2}}\right)^{-3} = \left(\frac{2x^{-6+6}}{y^{-2}}\right)^{-3} = \left(\frac{2x^0}{y^{-2}}\right)^{-3} = \left(\frac{2(1)}{y^{-2}}\right)^{-3} = \left(\frac{2}{y^{-2}}\right)^{-3} = \frac{2^{-3}}{y^{(-2)(-3)}} = \frac{1}{2^3y^6} = \frac{1}{8y^6}$$

Example: Simplify $\frac{(x^2-y)(x^2+y)^5}{(x^2+y)^{-2}(x^2+y)^6}$

Simplify the three factors with the same base

$$\begin{aligned}\frac{(x^2-y)(x^2+y)^5}{(x^2+y)^{-2}(x^2+y)^6} &= (x^2-y)(x^2+y)^{5-(-2)-6} = (x^2-y)(x^2+y) \\ &= x^2x^2 + x^2y - x^2y - y \cdot y = x^4 - y^2\end{aligned}$$

Practice: Simplify the following.

a) $\left(\frac{x+y}{3z}\right)^0$

d) $\frac{(2x^{-1/4})^2}{(4x)^{1/2}}$

g) $7^{x-3}7^x$

b) $\frac{(4a^5)(2a^{-6})}{8a^2}$

e) $(2x^3+y^{-2})(x-3y^7)$

h) $\left(\left(\frac{x^r}{y^t}\right)^2\left(\frac{x^{2r}}{y^{4t}}\right)^{-2}\right)^{-3}$

c) $\frac{1}{(2x+y)^{-3}}$

f) $(a^6b^2)^{-1} \div (a^{-3}b)^2$

f) $24\sqrt{x}\sqrt{y^3} \div 12\sqrt{x^3}\sqrt{y}$

Answers:

a) 1 b) $\frac{1}{a^3}$ c) $(2x+y)^3$ d) $\frac{2}{x}$ e) $2x^4 - 6x^3y^7 + xy^2 - 3y^5$ f) $\frac{1}{b^4}$ g) 7^{2x-3} h) $x^{6r}y^{-18t}$ i) $\frac{2y}{x}$

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