Counting rules for probabilities

The probability of an event is given by:

$$P(\text{event}) = \frac{\text{number of outcomes in event}}{\text{total number of outcomes}}$$

At times it can be tricky to find the number of outcomes. Here are some useful counting rules.

Summary of counting rules

Situation	Formula	Example
Multi-stage experiment k_1 ways for stage 1, k_2 ways for stage 2, ..., k_r ways for stage r	$k_1 \cdot k_2 \cdot k_3 \cdots k_r$	Two dice are tossed. Total outcomes: $(6)(6) = 36$ You own 4 jeans, 12 shirts, 4 sneakers. Total outfits: $(4)(12)(4) = 192$
Permutations (order matters)		
Arranging r objects from n	$\frac{n!}{(n-r)!}$	Committee of 8 chooses president, VP, secretary: $\frac{8!}{5!} = (8)(7)(6) = 336$
Arranging all n objects	$n!$	Visit 6 cities in order: $6! = (6)(5)(4)(3)(2)(1) = 720$
Combinations (order doesn't matter)		
Choosing r objects from n	$\binom{n}{r} = \frac{n!}{r!(n-r)!}$	Choose 4 cards from 52-card deck: $\binom{52}{4} = \frac{52!}{4! \cdot 48!} = 270{,}725$

Example: Password generation

A computer system requires passwords to contain at least one digit. If four characters are generated at random, and each is equally likely to be any of the 26 letters or 10 digits, what is the probability that a valid password will be generated?

Solution: Use the complement: $P(\text{at least one digit}) = 1 - P(\text{no digits})$.

Total ways to choose 4 characters: 36^4 (since $26 + 10 = 36$ options per character)

Ways to choose 4 characters with no digits: 26^4

$$P(\text{at least one digit}) = \frac{36^4 - 26^4}{36^4} = 1 - \left(\frac{26}{36}\right)^4 \approx 0.73$$

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Example: Committee selection

A group of 12 people consists of 5 men and 7 women. A 5-person committee is selected at random. Find the probability that the committee consists of:

Total ways to select 5 people from 12: $\binom{12}{5} = \frac{12!}{5! \cdot 7!} = 792$

a) Three men (and two women)

$$P(\text{3 men}) = \frac{\binom{5}{3} \cdot \binom{7}{2}}{\binom{12}{5}} = \frac{(10)(21)}{792} = \frac{210}{792} \approx 0.27$$

b) One man and two women (Note: This selects only 3 people, so we need 2 more from remaining)

If the question means exactly 1 man total in the 5-person group:

$$P(\text{1 man, 4 women}) = \frac{\binom{5}{1} \cdot \binom{7}{4}}{\binom{12}{5}} = \frac{(5)(35)}{792} = \frac{175}{792} \approx 0.22$$

c) At least one man

Use the complement: $P(\text{at least 1 man}) = 1 - P(\text{no men})$

$$P(\text{at least 1 man}) = 1 - \frac{\binom{7}{5}}{\binom{12}{5}} = 1 - \frac{21}{792} = \frac{771}{792} \approx 0.97$$