Finding the Determinant by Cofactor Expansion

Step 1: Choose any row or column in a given matrix, preferably the one that contains most zeros

Step 2: Along that row or column, multiply each entry by the corresponding cofactor $(-1)^{i+j} \det(A_{ij})$ where A_{ij} is a matrix obtained by deleting row i and column j from matrix A and then add the results.

Note: To find a determinant for a 2 x 2 matrix, you multiply entries on the main diagonal and subtract the product of entries on the opposite diagonal.

Example: To find the determinant of $\begin{bmatrix} 2 & 3 & 7 \\ -4 & 0 & 6 \\ 1 & 5 & 0 \end{bmatrix}$ using cofactor expansion we will choose last row.

$$\begin{vmatrix} 2 & 3 & 7 \\ -4 & 0 & 6 \\ 1 & 5 & 0 \end{vmatrix} = 1(-1)^{3+1}\begin{vmatrix} 3 & 7 \\ 0 & 6 \end{vmatrix} + 5(-1)^{3+2}\begin{vmatrix} 2 & 7 \\ -4 & 6 \end{vmatrix} + 0(-1)^{3+3}\begin{vmatrix} 2 & 3 \\ -4 & 0 \end{vmatrix}$$

$$= 1(1)[3(6) - 0(7)] + 5(-1)[2(6) - (-4)(7)] + 0$$

$$= -182$$

Question 1: Fill in the missing solution.

$$\begin{vmatrix} 3 & 1 & -5 \\ -1 & 3 & 0 \\ 7 & -4 & 2 \end{vmatrix} = (\quad)(-1)^{[\quad]}\begin{vmatrix} \quad & \quad \\ \quad & \quad \end{vmatrix} + (\quad)(-1)^{[\quad]}\begin{vmatrix} \quad & \quad \\ \quad & \quad \end{vmatrix} + (\quad)(-1)^{[\quad]}\begin{vmatrix} \quad & \quad \\ \quad & \quad \end{vmatrix}$$

$$=$$

Answer: _____

Question 2: Fill in the missing solution.

$$\begin{vmatrix} 2 & 6 & 4 \\ -3 & 1 & 5 \\ 7 & -1 & 2 \end{vmatrix} = (\quad)(-1)^{[\quad]}\begin{vmatrix} \quad & \quad \\ \quad & \quad \end{vmatrix} + (\quad)(-1)^{[\quad]}\begin{vmatrix} \quad & \quad \\ \quad & \quad \end{vmatrix} + (\quad)(-1)^{[\quad]}\begin{vmatrix} \quad & \quad \\ \quad & \quad \end{vmatrix}$$

$$=$$

Answer: _____

Question 3: Find the determinants of the following.

a) $\begin{bmatrix} -3 & 7 & 0 \\ 2 & 5 & 5 \\ -1 & 1 & 0 \end{bmatrix}$

Answer: _____

b) $\begin{bmatrix} 3 & 3 & 1 \\ -1 & -3 & -4 \\ 1 & 0 & 5 \end{bmatrix}$

Answer: _____

c) $\begin{bmatrix} 2 & 3 & 1 \\ -8 & 0 & 30 \\ 4 & 0 & 0 \end{bmatrix}$

Answer: _____

Answers:

Q1) 105

Q2) 244

Q3) a. - 20

Q3) b. - 39

Q3) c. 360

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