Calculating Probabilities

Assuming equally likely outcomes, the **probability** of an event can be calculated as

$$p(\text{event}) = \frac{\text{number of outcomes in event}}{\text{total number of outcomes}}$$

Example: The following table summarizes the numbers of defective and nondefective medical devices produces by two plants:

	Plant A	*Plant B*
Defective	*20*	*30*
Nondefective	*180*	*70*

a) The probability of selecting a defective device is

$$p(\text{defective}) = \frac{\text{Number of defective devices}}{\text{Total number of devices}} = \frac{20 + 30}{20 + 30 + 180 + 70} = \frac{50}{300} = 0.17$$

b) The probability of selecting a device produced by plant B is

$$p(\text{plant B}) = \frac{\text{Number of devices from plant B}}{\text{Total number of devices}} = \frac{30 + 70}{300} = \frac{100}{300} = 0.33$$

c) The probability of selecting a defective device from plant B

$$p(\text{defective} \cap \text{plant B}) = \frac{\text{Number of defective devices that are from plant B}}{\text{Total number of devices}} = \frac{30}{300} = 0.1$$

Some useful Probability Properties

The probability of an event A not occurring is given by

NOT A $P(\bar{A}) = 1 - P(A)$

The probability of either event A or event B occurring is given by

A or B $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

If the events A and B are mutually exclusive (cannot both occur at the same time), then

$P(A \cap B) = 0$ and $P(A \cup B) = P(A) + P(B)$

Remember:

The sum of the probabilities of all possible outcomes is 1

The probability of each outcome is a number between 0 and 1

Example: Being married and being over 30 years old are ***not mutually exclusive*** events.
Being a teenager and being over 30 years old ***are mutually exclusive*** events.

The **conditional probability** of event A, given that event B has occurred is

A given B $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$ or rearranging $P(A \cap B) = P(A \mid B)\,P(B)$

If two events A and B are **independent** (the occurrence of one event does not influence the probability of the other), then

$P(A \mid B) = P(A)$ and $P(A \cap B) = P(A)P(B)$

Example: Consider the table below with the probabilities as shown:

	Male	*Female*
Colourblind	*0.04*	*0.002*
Not colourblind	*0.47*	*0.488*

Find:

a) The probability of selecting a colourblind male

P (Male $\cap$ colorblind) = 0.04 (Taken directly from the table)

b) The probability of selecting a non colourblind female

P (Not colourblind $\cap$ female) = 0.488 (Taken directly from the table)

c) The probability of selecting a male

0.04+0.47=0.51

Explanation: A male is colourblind ***or*** not colour blind and since events 'colourblind' and 'not colourblind' are mutually exclusive, we can use $P(A \cup B) = P(A) + P(B)$

d) The probability of selecting a colour blind person

0.04+0.002=0.042

e) The probability of selecting either a male ***or*** a colourblind person

P (male $\cup$ colourblind) = P (male) + P(colourblind) - P(male $\cap$ colourblind)

= 0.51 + 0.042 - 0.04

= 0.512

f) The probability of being colourblind, ***if*** the person is male:

Given

$$P(\text{colourblind} \mid \text{male}) = \frac{P(\text{colourblind} \cap \text{male})}{P(\text{male})}$$

$$= \frac{0.04}{0.51}$$

= 0.078

g) Determine if two events 'being a male' and 'being colourblind' are independent.

If two events are **independent**, then $P(A \cap B) = P(A)P(B)$

P (colourblind $\cap$ male) = 0.04

P (colourblind) P (male)= (0.042) (0.51) = 0.02

Not equal; therefore, dependent

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