Ratios and proportions

What is a ratio?

A ratio is a comparison of two or more quantities. Ratios can be written in several equivalent ways:

Colon Notation	Fraction Notation	Word Notation
$3:4$	$\frac{3}{4}$	3 to 4

Ratios should be expressed in lowest terms by dividing all parts by their greatest common factor (GCF).

Example: Express the ratio $24:36$ in lowest terms.

GCF of 24 and 36 is 12

$$24:36 = (24 \div 12):(36 \div 12) = 2:3$$

Ratios with more than two terms

Ratios can compare three or more quantities. Simplify by dividing all terms by the GCF.

Example: A company allocates its budget to marketing, operations, and R&D in the ratio $5:3:2$. If the total budget is $500,000, how much goes to each department?

$$\text{Total parts} = 5 + 3 + 2 = 10 \text{ parts}$$

$$\text{Value of 1 part} = \$500{,}000 \div 10 = \$50{,}000$$

$$\text{Marketing: } 5 \times \$50{,}000 = \$250{,}000$$

$$\text{Operations: } 3 \times \$50{,}000 = \$150{,}000$$

$$\text{R\&D: } 2 \times \$50{,}000 = \$100{,}000$$

What is a proportion?

A proportion is a statement that two ratios are equal.

$$\frac{a}{b} = \frac{c}{d} \quad \text{or equivalently} \quad a:b = c:d$$

In a proportion, the **cross-products** are equal: $a \times d = b \times c$

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Solving proportions

To find an unknown value in a proportion, use cross-multiplication.

Cross-Multiplication Rule	Example
If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$	$\frac{x}{5} = \frac{12}{20} \implies 20x = 60 \implies x = 3$

Example: Solve for x: $\frac{7}{x} = \frac{21}{45}$

$$7 \times 45 = 21 \times x$$

$$315 = 21x$$

$$x = 315 \div 21 = 15$$

Example: If 12 items cost $45, how much do 20 items cost?

$$\frac{12 \text{ items}}{\$45} = \frac{20 \text{ items}}{\$x}$$

$$12x = 45 \times 20 = 900$$

$$x = \$75$$

Caution: When setting up a proportion, ensure the same units are in corresponding positions. Keep items with items and dollars with dollars on each side.

Direct vs. inverse proportion

Direct proportion: As one quantity increases, the other increases proportionally.

$$\frac{a_1}{b_1} = \frac{a_2}{b_2}$$

Inverse proportion: As one quantity increases, the other decreases. The products are constant.

$$a_1 \times b_1 = a_2 \times b_2$$

Example: If 8 workers can complete a project in 15 days, how many workers are needed to complete it in 10 days?
This is inverse proportion—more workers means fewer days.

$$8 \times 15 = x \times 10$$

$$120 = 10x$$

$$x = 12 \text{ workers}$$

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