Linear inequalities

Solving linear inequalities

Solving linear inequalities is similar to solving linear equations except we need to be careful with the direction of the inequality sign. The solution will be a set of values of x.

Important: If there is division or multiplication by a negative number, you must flip the inequality sign.

Example: Solve $1 - 2x \leq 7$.

$$\begin{aligned} 1 - 2x &\leq 7 \\ -2x &\leq 6 \\ \frac{-2x}{-2} &\geq \frac{6}{-2} \quad \text{(flip the sign)} \\ x &\geq -3 \end{aligned}$$

This can be written in interval notation as $[-3, \infty)$.

Interval notation

Inequality	In words	Interval notation
$x < a$	x is less than a	$(-\infty, a)$
$x \leq a$	x is less than or equal to a	$(-\infty, a]$
$x > a$	x is greater than a	(a, ∞)
$x \geq a$	x is greater than or equal to a	$[a, \infty)$
$a < x < b$	x is between a and b (exclusive)	(a, b)
$a \leq x \leq b$	x is between a and b (inclusive)	$[a, b]$
$a < x \leq b$	x is greater than a and less than or equal to b	$(a, b]$
$a \leq x < b$	x is greater than or equal to a and less than b	$[a, b)$

Notation tips:

- Parentheses $(\ )$ mean the endpoint is **not included** (open circle on number line)
- Brackets $[\ ]$ mean the endpoint **is included** (closed circle on number line)
- Infinity (∞) always uses a parenthesis since it is not a number

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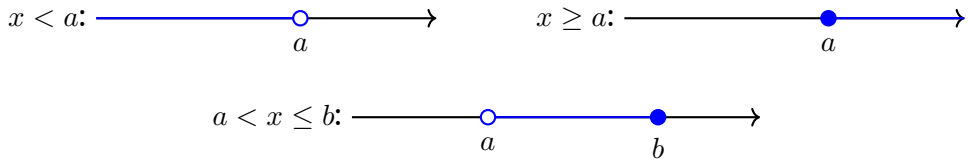
Graphing on a number line



Compound inequalities

Compound inequalities combine two inequalities using **AND** or **OR**.

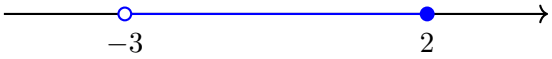
AND ($\cap$, intersection): The solution contains only values that satisfy *both* inequalities. This is the overlap of the two sets.

OR ($\cup$, union): The solution contains values that satisfy *either* inequality. This includes all elements from both sets.

Example: Solve $x > -3$ AND $x \leq 2$.

The solution must satisfy both conditions: values greater than -3 that are also at most 2.

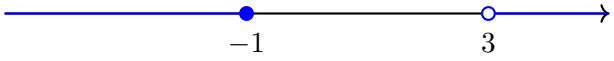
Solution: $(-3, 2]$



Example: Solve $x \leq -1$ OR $x > 3$.

The solution satisfies at least one condition: values at most -1, or values greater than 3.

Solution: $(-\infty, -1] \cup (3, \infty)$



Caution: When solving compound inequalities, solve each part separately, then combine the solutions using intersection (AND) or union (OR).

Practice problems. Solve each inequality and write the answer in interval notation.

1. $3x - 5 \leq 10$	**2.** $7 - 4x > 15$	**3.** $5x + 8 \geq 3$	**4.** $-6x + 4 < 16$
5. $\frac{x}{3} > 2$	**6.** $\frac{x}{-2} \leq 4$	**7.** $\frac{x+5}{2} < 3$	**8.** $\frac{x-4}{-3} \geq 2$
9. $x > 1$ AND $x \leq 6$	**10.** $x < -4$ OR $x \geq 2$	**11.** $-3 \leq x < 4$	**12.** $-2(x-3) \geq 8$

Answers: 1. $(-\infty, 5]$ 2. $(-\infty, -2)$ 3. $[-1, \infty)$ 4. $(-2, \infty)$ 5. $(6, \infty)$ 6. $[-8, \infty)$
7. $(-\infty, 1)$ 8. $(-\infty, -2]$ 9. $(1, 6]$ 10. $(-\infty, -4) \cup [2, \infty)$ 11. $[-3, 4)$ 12. $(-\infty, -1]$

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