# Linear inequalities

## Solving linear inequalities

Solving linear inequalities is similar to solving linear equations except we need to be careful with the direction of the inequality sign. The solution will be a set of values of $x$.

Important: If there is division or multiplication by a negative number, you must flip the inequality sign.

Example: Solve $1 - 2x \leq 7$.

$$1 - 2x \leq 7$$
$$-2x \leq 6$$
$$\frac{-2x}{-2} \geq \frac{6}{-2} \quad \text{(flip the sign)}$$
$$x \geq -3$$

This can be written in interval notation as $[-3, \infty)$.

## Interval notation

| Inequality | In words | Interval notation |
|---|---|---|
| $x < a$ | $x$ is less than $a$ | $(-\infty, a)$ |
| $x \leq a$ | $x$ is less than or equal to $a$ | $(-\infty, a]$ |
| $x > a$ | $x$ is greater than $a$ | $(a, \infty)$ |
| $x \geq a$ | $x$ is greater than or equal to $a$ | $[a, \infty)$ |
| $a < x < b$ | $x$ is between $a$ and $b$ (exclusive) | $(a, b)$ |
| $a \leq x \leq b$ | $x$ is between $a$ and $b$ (inclusive) | $[a, b]$ |
| $a < x \leq b$ | $x$ is greater than $a$ and less than or equal to $b$ | $(a, b]$ |
| $a \leq x < b$ | $x$ is greater than or equal to $a$ and less than $b$ | $[a, b)$ |

**Notation tips:**

- Parentheses ( ) mean the endpoint is **not included** (open circle on number line)

- Brackets [ ] mean the endpoint **is included** (closed circle on number line)

- Infinity ($\infty$) always uses a parenthesis since it is not a number

# Graphing on a number line

$x < a$:   

$x \geq a$:   

$a < x \leq b$:   
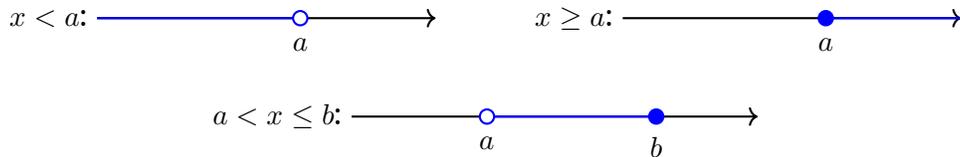
# Compound inequalities

Compound inequalities combine two inequalities using **AND** or **OR**.

**AND** ($\cap$, intersection): The solution contains only values that satisfy *both* inequalities. This is the overlap of the two sets.

**OR** ($\cup$, union): The solution contains values that satisfy *either* inequality. This includes all elements from both sets.

Example: Solve $x > -3$ AND $x \leq 2$.

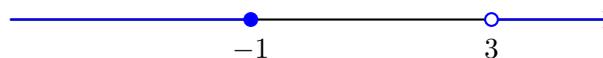The solution must satisfy both conditions: values greater than $-3$ that are also at most $2$.

Solution: $(-3, 2]$



Example: Solve $x \leq -1$ OR $x > 3$.

The solution satisfies at least one condition: values at most $-1$, or values greater than $3$.

Solution: $(-\infty, -1] \cup (3, \infty)$



Caution: When solving compound inequalities, solve each part separately, then combine the solutions using intersection (AND) or union (OR).

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