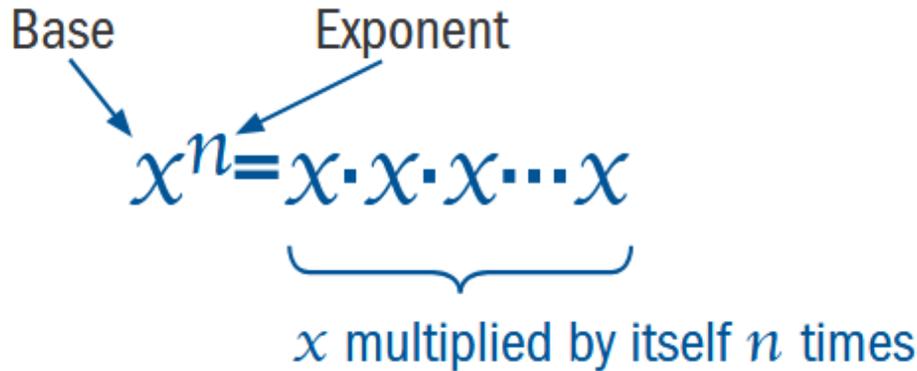


# Exponents



When an exponent is a positive integer, it indicates how many times the base is multiplied by itself.



Example:

$$2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$$

When using a calculator, the exponent key might be  $\wedge$  or  $x^y$ .

## 1 Properties of exponents

Property	Example
Zero exponent: $x^0 = 1$ if $x \neq 0$	$5^0 = 1$
Negative exponent: $x^{-n} = \frac{1}{x^n}$	$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$
Product of like bases: $x^m x^n = x^{m+n}$	$a^3 a^4 = a^{3+4} = a^7 \leftarrow a^3 a^4 = (aaa)(aaaa) = a^7$
Quotient of like bases: $\frac{x^m}{x^n} = x^{m-n}$	$\frac{y^{11}}{y^3} = y^{11-3} = y^8$
Power of a power: $(x^m)^n = x^{mn}$	$(x^4)^5 = x^{4 \cdot 5} = x^{20}$
Power of a product: $(xy)^n = x^n y^n$	$(st)^6 = s^6 t^6$
Power of a quotient: $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$	$\left(\frac{x}{y}\right)^7 = \frac{x^7}{y^7}$
$N$ th root: $x^{\frac{1}{N}} = \sqrt[N]{x}$	$x^{\frac{1}{2}} = \sqrt{x}$ $x^{\frac{1}{3}} = \sqrt[3]{x}$
Fractional exponent: $x^{\frac{m}{n}} = \sqrt[n]{x^m}$	$16^{\frac{3}{4}} = \sqrt[4]{16^3} = (\sqrt[4]{16})^3 = 2^3 = 8$
Adding like-terms: $ax^n + bx^n = (a+b)x^n$	$3x^2 + 8x^2 = 11x^2$

**Caution:**  $(x + y)^n \neq x^n + y^n$

**Example:**

$$\left(\frac{2x^{-6}x^6}{y^{-2}}\right)^{-3} = \left(\frac{2x^{-6+6}}{y^{-2}}\right)^{-3} = \left(\frac{2x^0}{y^{-2}}\right)^{-3} = \left(\frac{2(1)}{y^{-2}}\right)^{-3} = \left(\frac{2}{y^{-2}}\right)^{-3} = \frac{2^{-3}}{y^{(-2)(-3)}} = \frac{1}{2^3y^6} = \frac{1}{8y^6}$$

**Example:**

Simplify the three factors with the same base:

$$\frac{(x^2 - y)(x^2 + y)^5}{(x^2 + y)^{-2}(x^2 + y)^6} = (x^2 - y)(x^2 + y)^{5 - (-2) - 6} = (x^2 - y)(x^2 + y) = x^2x^2 + x^2y - x^2y - y \cdot y = x^4 - y^2$$

Practice: Simplify the following.

a)  $\left(\frac{x + y}{3z}\right)^0$

b)  $\frac{(4a^5)(2a^{-6})}{8a^2}$

c)  $\frac{1}{(2x + y)^{-3}}$

d)  $\frac{(2x^{-\frac{1}{4}})^2}{(4x)^{\frac{1}{2}}}$

e)  $(2x^3 + y^{-2})(x - 3y^7)$

f)  $(a^6b^2)^{-1} \div (a^{-3}b)^2$

g)  $7^{x-3}7^x$

h)  $\left(\left(\frac{x^r}{y^t}\right)^2 \left(\frac{x^{2r}}{y^{4t}}\right)^{-2}\right)^{-3}$

i)  $24\sqrt{x}\sqrt{y^3} \div 12\sqrt{x^3}\sqrt{y}$

Answers:

a) 1   b)  $\frac{1}{a^3}$    c)  $(2x + y)^3$    d)  $\frac{2}{x}$    e)  $2x^4 - 6x^3y^7 + xy^{-2} - 3y^5$    f)  $\frac{1}{b^4}$    g)  $7^{2x-3}$    h)  $x^{6r}y^{-18t}$    i)  $\frac{2y}{x}$