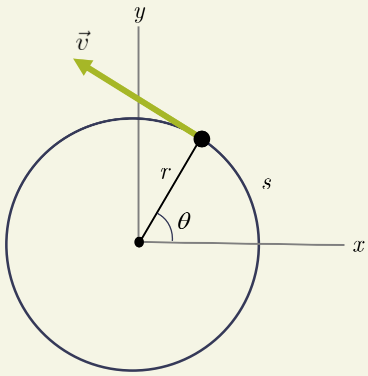
3 Kinematics of Circular Motion

Circular Motion

- The position of an object undergoing circular motion can be specified by the *radius* r and the *angular position* θ. The distance traveled by the object is given by

$$s = \theta r$$

if θ is measured in radians.



- The *angular velocity* of the object is

$$\omega = \frac{d\theta}{dt}.$$

By convention, ω is positive if the object is traveling counter-clockwise around the circle. The linear velocity of the object is always tangential to the circle and has a magnitude

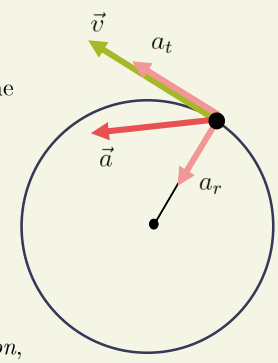
$$v = \omega r$$

as long as ω is measured in radians.

- The *angular acceleration* of the object is

$$\alpha = \frac{d\omega}{dt}.$$

An object is speeding up if α and ω have the same sign. The linear acceleration of the object can be decomposed into the *radial acceleration*,



$$a_r = \frac{v^2}{r} = \omega^2 r$$

which points to the centre of the circle and is related to the change of *direction* of the object, and the *tangential acceleration*,

$$a_t = \alpha r$$

which is related to the change in *speed* of the object. The total acceleration is

$$a = \sqrt{a_r^2 + a_t^2}.$$

Model: Uniform Circular Motion

If ω is constant (so $\alpha = 0$) then the object has a uniform speed given by

$$v = \frac{2\pi r}{T}$$

where T is the *period* of the motion – how long it takes to travel one full revolution.

An object in uniform circular motion still has *radial acceleration* (sometimes called *centripetal acceleration*), associated with the change in direction of the velocity. The radial acceleration points toward the centre of the circle and has magnitude

$$a_r = \frac{v^2}{r} = \omega^2 r$$

Model: Constant Angular Acceleration

If the object changes speed as it travels in a circle it will have both *radial* and *tangential acceleration*. If the object's angular acceleration α is constant, we can model the motion with the *rotational kinematic equations*,

$$\theta_f = \theta_i + \omega_i \Delta t + \frac{1}{2}\alpha \Delta t^2$$

$$\omega_f = \omega_i + \alpha \Delta t$$

$$\omega_f^2 = \omega_i^2 + 2\alpha \Delta\theta$$

These equations are analogous to the one dimensional straight line kinematic equations and can be used similarly.