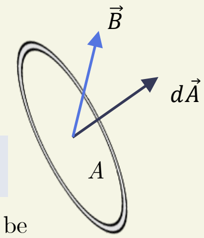
9 Faraday's Law and Inductance

Magnetic Flux

Magnetic flux is, loosely speaking, how much of the magnetic field $\vec{B}$ "flows through" a loop of area A. Mathematically, the flux is

$$\Phi_m = \int_{\text{loop}} \vec{B} \cdot d\vec{A}$$



If the magnetic field is uniform, then the flux can be written

$$\Phi_m = \vec{B} \cdot \vec{A}.$$

The units of magnetic flux are webers, 1 Wb = 1 Tm^2.

Faraday's Law

An emf is induced in a closed loop (or coil of N turns) if the magnetic flux through the loop *changes*:

$$\mathcal{E} = N \left| \frac{d\Phi_m}{dt} \right|$$

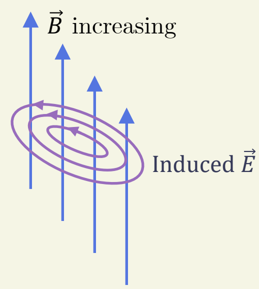
The magnetic flux can change either when the loop changes size or rotates (called *motional emf*), or when the magnetic field changes.

The direction of the induces emf is given by *Lenz's law*: The current created by the induced emf has a direction such that the induced magnetic field opposes the *change* in flux.

Induced Electric Fields

Faraday's law implies that *a changing magnetic field induces an electric field*. This relationship is given by

$$\oint \vec{E} \cdot d\vec{s} = A \left| \frac{dB}{dt} \right|$$



This induced electric field is a little different from the one we're used to:

- It's this *induced* electric field that drives the current in a loop when the magnetic field changes.
- An electric field created by *charges* – called a Coulomb electric field – is conservative, but this induced field is not.
- The field lines are always loops.
- This electric field has no associated potential.

Inductors

An inductor is a circuit element that consists of a coil of wire; a *solenoid* is an example of an inductor. An ideal inductor has no resistance in the wire.

The *inductance* L of a coil is defined to be the magnetic flux through the coil divided by the current through the wire:

$$L = \frac{\Phi_m}{I}$$

The units of inductance are henries, 1 H = 1 Tm^2/A.

The inductance of a *solenoid* of N turns, area A, and length l is

$$L_{\text{solenoid}} = \frac{\mu_0 N^2 A}{l}.$$

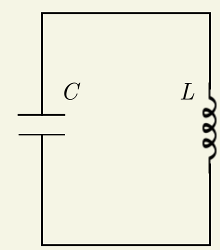
In a circuit, the inductor has a voltage

$$\Delta V_L = -L\frac{dI}{dt}.$$

LC Circuits

A circuit consisting of an inductor (L) and a capacitor (C) is an *electric oscillator* with frequency $f = \omega/2\pi$, where the angular frequency is

$$\omega = \sqrt{\frac{1}{LC}}$$



The charge on the capacitor oscillates according to

$$Q(t) = Q_0 \cos \omega t,$$

and the current through the inductor as

$$I(t) = I_{\text{max}} \sin \omega t,$$

where $I_{\text{max}} = \omega Q_0$.

Energy in an Inductor

Inductors store an amount of energy given by

$$U_L = \frac{1}{2}LI^2$$

Like the electric field, the magnetic field stores energy. The energy density of a magnetic field is

$$u_B = \frac{1}{2\mu_0}B^2.$$

LR Circuits

The current in an LR circuit decays exponentially with time:

$$I(t) = I_0 e^{-t/\tau}$$

where the *time constant* is $\tau = L/R$.