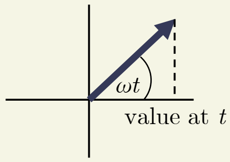
9 AC Circuits

AC Sources and Phasors

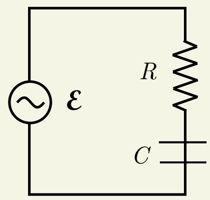
An alternating current (AC) generator produces an emf that oscillates sinusoidally in time with an angular frequency $\omega = 2\pi f$ and peak emf $\mathcal{E}_0$:

$$\mathcal{E} = \mathcal{E}_0 \cos \omega t$$



The emf, voltage across circuit elements, and current through circuit elements all oscillate in time and can be represented by a *phasor*. A phasor is a vector that rotates counterclockwise about the origin at angular frequency ω. The instantaneous value of the quantity is the projection of the phasor on the x-axis.

RC Filter Circuits



The peak current in an RC circuit is

$$I = \frac{\mathcal{E}_0}{\sqrt{R^2 + X_C^2}},$$

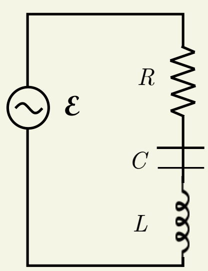
and the voltages across the resistor and capacitor are

$$V_R = \frac{\mathcal{E}_0 R}{\sqrt{R^2 + X_C^2}} \quad \text{and} \quad V_C = \frac{\mathcal{E}_0 X_C}{\sqrt{R^2 + X_C^2}}$$

An RC circuit can act as a frequency filter:

- The voltage across the capacitor $V_C \to \mathcal{E}_0$ as $\omega \to 0$. Connecting an element across the capacitor allows it to act as a *low-pass filter*: only low frequencies will be transmitted.
- The voltage across the resistor $V_R \to \mathcal{E}_0$ as $\omega \to \infty$. Connecting an element across the resistor allows it to act as a *high-pass filter*: only high frequencies will be transmitted.

The Series RLC Circuit



The peak current in an RLC circuit is

$$I = \frac{\mathcal{E}_0}{Z}$$

where Z is the impendence:

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

The peak voltages are $V_R = IR$, $V_C = IX_C$, and $V_L = IX_L$. The emf and current are *out of phase* by the *phase angle*

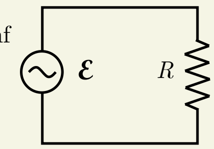
$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right).$$

The maximum current $I_{max} = \mathcal{E}_0/R$ through the circuit occurs at the *resonance frequency*

$$\omega = \frac{1}{\sqrt{LC}}.$$

One Element Circuits

Resistor Circuit

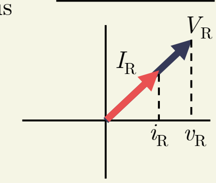


A single resistor R connected to an oscillatory emf source $\mathcal{E}$ has an instantaneous voltage given by

$$v_R = V_R \cos \omega t$$

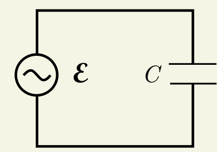
where V_R is the *peak voltage*. The instantaneous current through the resistor is

$$i_R = I_R \cos \omega t$$



where I_R is the *peak current*. The current through and voltage across the resistor are *in phase*; the peaks occur at the same time.

Capacitor Circuit

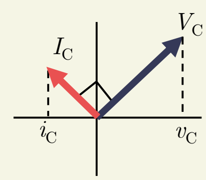


A single capacitor C connected to an oscillatory emf source $\mathcal{E}$ has an instantaneous voltage given by

$$v_C = V_C \cos \omega t.$$

The instantaneous current through the resistor is

$$i_C = I_C \cos(\omega t + \tfrac{\pi}{2}),$$



where $I_C = \omega C V_C$ is the peak current. The current through and voltage across the capacitor are *out of phase*; the current leads the voltage by $\pi/2$ (90°).

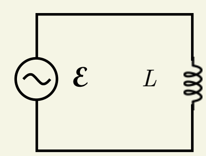
The capacitive reactance X_C is defined to be

$$X_C = \frac{1}{\omega C}$$

and has units of ohms. The peak current can be written as

$$I_C = \frac{V_C}{X_C}$$

Inductor Circuit

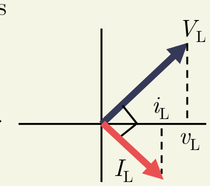


A single inductor L connected to an oscillatory emf source $\mathcal{E}$ has an instantaneous voltage given by

$$v_L = V_L \cos \omega t$$

The instantaneous current through the resistor is

$$i_L = I_L \cos(\omega t - \tfrac{\pi}{2})$$



where $I_L = V_L/\omega L$ is the peak current. The current through and voltage across the capacitor are *out of phase*; the current lags the voltage by $\pi/2$ (90°).

The inductive reactance X_L is defined to be

$$X_L = \omega L$$

and has units of ohms. The peak current can be written as

$$I_L = \frac{V_L}{X_L}$$

Power

The emf supplies energy to the circuit at the rate

$$P_{source} = I_{rms}\mathcal{E}_{rms} \cos \phi,$$

where $I_{rms} = I_R/\sqrt{2}$, $\mathcal{E}_{rms} = \mathcal{E}_0/\sqrt{2}$, and ϕ is the phase factor.

The *average* power dissipated by a resistor is

$$P_R = I_{rms}^2 R = \frac{V_{rms}^2}{R} = I_{rms} V_{rms}$$

The average power dissipated by a capacitor or inductor is *zero*.

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