9 Work and the Energy Principle

The Dot Product

The *dot* (or *scalar*) *product* of two vectors $\vec{A}$ and $\vec{B}$ is defined as

$$\vec{A} \cdot \vec{B} = AB\cos(\alpha),$$

where α is the angle between the vectors.

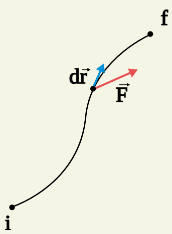
We can also write the dot product in terms of vector components in a coordinate system:

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y.$$

Work

Work is the transfer of energy between the environment and the system. The work done by a force $\vec{F}$ on a particle that undergoes a displacement from point i to point f is

$$W = \int_i^f \vec{F} \cdot d\vec{r}.$$



If the force is *constant*, then the integral simplifies and the work is just

$$W = \vec{F} \cdot \Delta\vec{r},$$

where $\Delta\vec{r} = \vec{r}_f - \vec{r}_i$.

If the work done is *positive* ($W > 0$), the energy is transferred *into* the system; if *negative* ($W < 0$) the energy is transferred *out* of the system.

Power

Power is the rate at which energy is transferred or transformed:

$$P = \frac{dE_{\text{sys}}}{dt}.$$

It has units of watts, where 1 W = 1 J/s.

If the energy is transferred by work done on the system, then the power can be calculated as

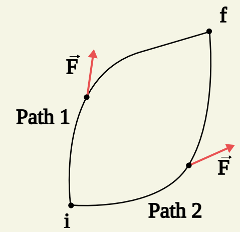
$$P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}.$$

Force and Potential Energy

A force for which the work done on a particle as it moves from an initial position to a final position is independent of the path followed is called a *conservative force*.

A potential energy can be associated with any conservative force, and is defined to be the negative of the work done between i and f:

$$\Delta U = -W(i \to f).$$



Examples of conservative forces include gravity, the spring force, and the electrostatic force; forces such as friction and tension are *not* conservative and have no potential energy associated with them.

Given the definition of work, we can write the relationship between potential energy and its associated conservative force as

$$\Delta U = -\int_i^f \vec{F} \cdot d\vec{r}.$$

We can also rearrange this for the force:

$$F_s = -\frac{dU}{ds},$$

where F_s is the component of the force along the s direction.

Thermal Energy and Dissipative Forces

The thermal energy of a system is the sum of all microscopic kinetic and potential energies:

$$E_{\text{th}} = K_{\text{micro}} + U_{\text{micro}},$$

and is related to the *temperature* of the system.

Forces that transform energy into thermal energy, such as friction or drag, are called *dissipative forces*. Dissipative forces always *increase* the thermal energy of a system.

In the case of friction, the increase in thermal energy can be calculated to be

$$\Delta E_{\text{th}} = f_k \Delta s,$$

where Δs is the distance through which the friction force acts.

The Energy Principle

The *energy principle* says that the total system energy can change through work done on the system by *external forces*:

$$\Delta E_{\text{sys}} = \Delta K + \Delta U + \Delta E_{\text{th}} = W_{\text{ext}}.$$

In an *isolated system*, $W_{\text{ext}} = 0$ and the total system energy is *conserved*. This is the *law of conservation of energy*.