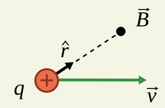
8 The Magnetic Field

Magnetic Field

Moving charges create magnetic fields. For a single point charge, the magnetic field is given by the *Biot-Savart law*,

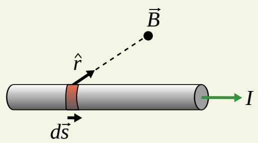
$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2},$$

where $\mu_0 = 4\pi \times 10^{-7}$ Tm/A is the *permeability constant.*



For a current segment of length $d\vec{s}$, the Biot-Savart law is

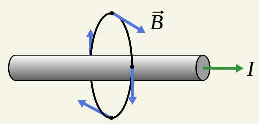
$$\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{s}\times\hat{r}}{r^2}.$$



Some common fields are:

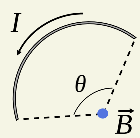
- A distance r from long straight wire carrying current I

$$B = \frac{\mu_0 I}{2\pi r}$$



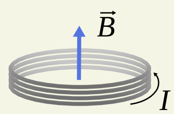
- At the centre of an θ radian arc of wire or radius R and carrying current I

$$B = \frac{\mu_0 I\theta}{4\pi R}$$



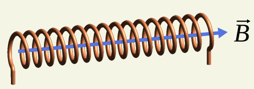
- At the centre of a loop of N turns and radius R, carrying current I

$$B = \frac{\mu_0 NI}{2R}$$



- Inside a solenoid of length ℓ and number of turns N, carrying current I

$$B = \frac{\mu_0 NI}{\ell}$$



Ampère's Law

Ampère's law states that the line integral of the magnetic field around a closed loop is proportional to the current passing through that loop:

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{\text{through}}.$$

This is useful for highly symmetric current distributions.

Magnetic Forces

Magnetic fields exert a *force* on *moving charges*:

$$\vec{F}_{\text{on } q} = q\vec{v}\times\vec{B}.$$

Cyclotron Motion
A charged particle moving perpendicular through a uniform magnetic field undergoes *cyclotron motion*, with radius r_{cyc} and frequency f_{cyc} given by

$$r_{\text{cyc}} = \frac{mv}{qB} \quad \text{and} \quad f_{\text{cyc}} = \frac{qB}{2\pi m}.$$

Force on a wire
Magnetic fields also exert a force on *current-carrying wires*:

$$\vec{F}_{\text{wire}} = I\vec{\ell}\times\vec{B},$$

where ℓ points in the direction of the current.

Force between wires
Two parallel wires of current I_1 and I_2, of length ℓ and separated by a distance d, will therefore exert a force on each other (attractive for current in the same direction, repulsive for opposite currents), with magnitude

$$F = \frac{\mu_0 \ell I_1 I_2}{2\pi d}.$$

Magnetic Dipoles

A current loop of area A is a magnetic dipole, with dipole moment

$$\vec{\mu} = (AI, \text{south to north pole}).$$

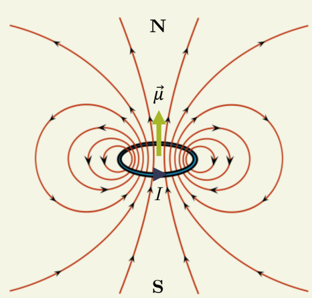
The magnetic field on the axis far from the loop at distance z is given by

$$\vec{B}_{\text{dipole}} = \frac{\mu_0}{4\pi}\frac{2\vec{\mu}}{z^3}.$$

A uniform magnetic field exerts no net force on the current loop, but will exert a torque

$$\vec{\tau} = \vec{\mu}\times\vec{B},$$

which tends to rotate the loop so that the dipole moment is aligned with the magnetic field.



Magnetism in Materials

Atomic electrons in a material have a magnetic dipole moment due to a property called spin. In some materials, called ferromagnetic, these dipole moments can align to create a strong magnetic field. The image to the right shows the domain structure of a ferromagnet; within each domain the dipole moments combine.