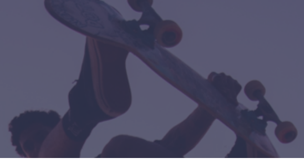
7 The Momentum Principle



Momentum

The *momentum* of a particle is defined as

$$\vec{p} = m\vec{v}.$$

Note that momentum has units of kg m/s and is a *vector* – it points in the same direction as the velocity $\vec{v}$.

We can write Newton's second law in terms of momentum:

$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt} = \frac{d\vec{p}}{dt}.$$

In words, this says that a force on an object changes the object's momentum.

The Momentum Principle

Now we can put these two new concepts to use. Starting from

$$\vec{F} = \frac{d\vec{p}}{dt},$$

multiple both sides by dt and integrate from $t = t_i$ to $t = t_f$:

$$\int_{t_i}^{t_f} \vec{F}\,dt = \int_{\vec{p}_i}^{\vec{p}_f} d\vec{p}.$$

The left hand side is the impulse $\vec{J}$, and the right hand side is $\Delta\vec{p} = \vec{p}_f - \vec{p}_i$. The *momentum principle* is a restatement of Newton's second law that says

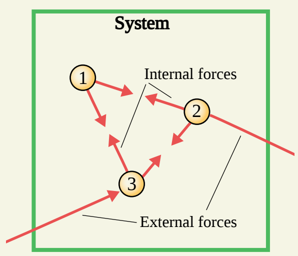
$$\Delta\vec{p} = \vec{J}.$$

An impulse delivered to an object changes its momentum.

Conservation of Momentum

For a system of N particles, the *total* momentum $\vec{P}$ (that's a capital P!) is the sum of the individual particle momenta:

$$\vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \cdots = \sum_{k=1}^{N} \vec{p}_k.$$



Thanks to Newton's third law, the sum of all the internal interaction forces is zero, so Newton's second law applies to the whole system,

$$\vec{F}_{\text{net}} = \frac{d\vec{P}}{dt},$$

where $\vec{F}_{\text{net}}$ is the sum of all *external forces* only.

Now, if the net external force is *zero*, $F_{\text{net}} = 0$, we call the system *isolated*. In that case, the total momentum of the system doesn't change – it is *conserved*:

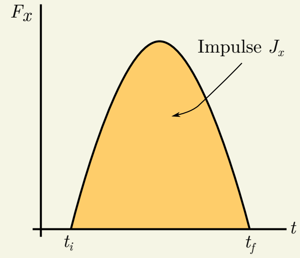
$$\text{LAW OF CONSERVATION OF MOMENTUM:} \quad \vec{P}_i = \vec{P}_f.$$

Impulse

The *impulse* is defined as

$$\vec{J} = \int_{t_i}^{t_f} \vec{F}(t)\,dt.$$

In a plot of the force (say, along the x-direction to makes things simpler) versus time, the impulse is the *area under the force curve* from time $t = t_i$ to $t = t_f$.

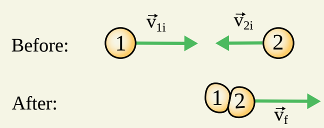


APPLICATION: Collisions

Inelastic Collisions

In an inelastic collision, two objects collide and stick together – they have a common final velocity. In an isolated system, momentum is conserved:

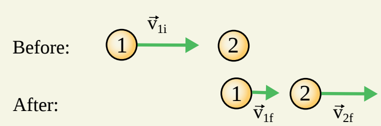
$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1 + m_2)\vec{v}_f.$$



Elastic Collisions

In an elastic collision, two objects collide and bounce off each other *elastically* – meaning the total energy is conserved. In the case where object 2 is at rest and the motion is along a straight line, the final speeds of each object are

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_{1i}; \quad v_{2f} = \frac{2m_1}{m_1 + m_2}v_{1i}.$$



APPLICATION: Explosions

In an explosion, one object breaks into two or more pieces. In an isolated system, momentum is conserved:

$$M_{\text{tot}}\vec{v}_i = m_1\vec{v}_{1f} + m_2\vec{v}_{2f} + \ldots$$