7 Current and Resistance

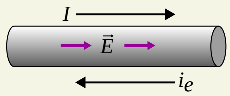
Current

Current is the motion of charges sustained by an electric field.

- Electron current i_e is the rate of flow of electrons.
- Conventional current I is the rate of flow of charge,

$$I = \frac{dQ}{dt}.$$

- The current density $J = I/A$, where A is the cross-sectional area of the wire.



Ohm's Law

An electric field E in a conductor creates a current density

$$J = \sigma E,$$

where σ is the *conductivity* of the material. The *resistivity* of the material is $\rho = 1/\sigma$. The resistance of a conductor of length L is given by

$$R = \frac{\rho L}{A}.$$

The unit of resistance is ohms ($1\ \Omega = 1$ V/A).
Ohm's law says that the current through a conductor is given by

$$I = \frac{\Delta V}{R},$$

where ΔV is the voltage across the ends of the conductor.

Kirchhoff's Laws

Kirchhoff's Junction Law

Since charge is conserved, the current into a junction is equal to the current leaving the junction:

$$\sum I_{\text{in}} = \sum I_{\text{out}}.$$

Kirchhoff's Loop Law

Since energy is conserved, the sum of all voltages in a complete loop in a circuit is zero:

$$\Delta V_{\text{loop}} = \sum_i \Delta V_i = 0.$$

The voltage here is $\Delta V = V_{\text{downstream}} - V_{\text{upstream}}$, so that

- For an ideal battery in the negative to positive direction, $\Delta V = +\mathcal{E}$.
- For an ideal battery in the positive to negative direction, $\Delta V = -\mathcal{E}$.
- For a resistor, the potential decreases across the resistor in the direction of the current, so $\Delta V = -IR$.

Batteries

A battery is a source of potential difference (voltage) ΔV_{bat}. This voltage causes an electric field in a connected wire, which establishes a current I. The magnitude of the current is determined by Ohm's law.

- An ideal battery has a voltage $\Delta V_{\text{bat}} = \mathcal{E}$.
- A real battery has internal resistance r and a voltage $\Delta V_{\text{bat}} = \mathcal{E} - Ir$.
- The power delivered by a battery is $P_{\text{bat}} = I\mathcal{E}$.

Resistor Circuits

Resistors in Series

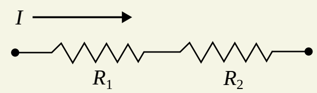
When connected in series, resistors have the same current through them but different voltage ΔV_R. Resistances add to an equivalent resistance:

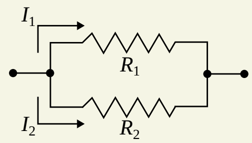
$$R_{\text{eq}} = R_1 + R_2 + R_3 + \cdots$$

Resistors in Parallel

When connected in parallel, resistors have the same voltage ΔV_R but different current. Resistances add in inverse to an equivalent resistance:

$$R_{\text{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots\right)^{-1}$$





Power

The power dissipated by a resistor is

$$P_R = I\Delta V_R = I^2 R = \frac{\Delta V_R^2}{R}.$$

Measuring Current and Voltage

- An *ammeter* measures current I in a circuit element. It must be placed in series with the element. An ideal ammeter has zero resistance.
- A voltmeter measures the voltage ΔV across a circuit element. It must be placed in parallel with the element. An ideal voltmeter has infinite resistance.

RC Circuits

A capacitor can be discharged by connecting it to a resistor, and charged by connecting it to a resistor and battery.

- Discharging a capacitor:

$$Q = Q_0 e^{-t/\tau} \qquad I = I_0 e^{-t/\tau}.$$

- Charging a capacitor:

$$Q = Q_0\left(1 - e^{-t/\tau}\right) \qquad I = I_0 e^{-t/\tau}.$$

The time constant $\tau = RC$ determines how quickly the capacitor is charged and discharged.