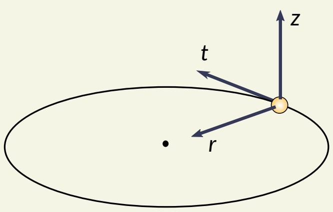
6 Dynamics of Circular Motion

The rtz Coordinate System

An x-y-z coordinate system is not great for dealing with circular motion. Better is the r-t-z coordinate system, where:

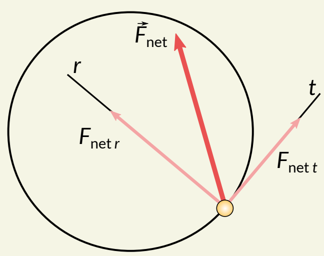
- the origin is at the particle's position,
- the r (radial) axis points from the particle to the centre of the circle,
- the t (tangential) axis points tangent to the circle in the counter-clockwise direction, and
- the z axis is perpendicular to the plane of motion.



Newton's Second Law for Circular Motion

Recall that for circular motion the acceleration has a radial (r) component related to the change in direction and a tangential (t) component related to the chnage in speed. In the rtz coordinate system, then, Newton's second law take the form

$$F_{\text{net }r} = ma_r = \frac{mv^2}{r} = m\omega^2 r$$
$$F_{\text{net }t} = ma_t = m\alpha r$$
$$F_{\text{net }z} = ma_z = 0$$



MODEL: Uniform Circular Motion

For *uniform circular motion*, $a_t = 0$, so Newton's second law is

$$F_{\text{net }r} = \frac{mv^2}{r} = m\omega^2 r$$
$$F_{\text{net }t} = 0$$
$$F_{\text{net }z} = 0$$

MODEL: Constant Angular Acceleration

For *nonuniform circular motion*, Newton's second law is

$$F_{\text{net }r} = m\omega^2 r$$
$$F_{\text{net }t} = m\alpha r$$
$$F_{\text{net }z} = 0$$

APPLICATION: Circular Orbits

In a circular orbit, the only force acting on the object (for example, the International Space Station or a satellite) is *gravity*. But we can't use the *flat Earth approximation* for this; instead, we model gravity as

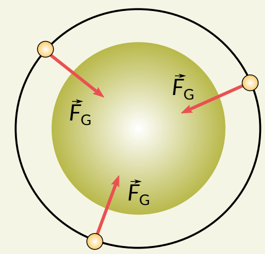
$$\vec{F}_G = (mg, \text{toward centre}).$$

Since this force is *radial* (or *central*), the orbit is uniform circular motion with acceleration g:

$$a_r = \frac{v_{\text{orbit}}^2}{r} = g.$$

The speed of the orbiting object is then

$$v_{\text{orbit}} = \sqrt{gr}.$$



APPLICATION: Roller Coasters

A roller coaster doing a loop-the-loop is in *nonuniform circular motion* – it slows down on its way up and speeds up on its way down. But at the top and bottom of the loop, the cart has only *radial* forces.

At the bottom, Newton's second law says

$$F_{\text{net }r} = n - mg = \frac{mv^2}{r},$$

or

$$n = mg + \frac{mv^2}{r},$$

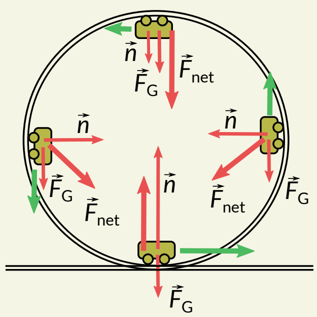
so you feel heavier than normal.

At the top, we have

$$F_{\text{net }r} = n + mg = \frac{mv^2}{r},$$

or

$$n = \frac{mv^2}{r} - mg.$$



When the normal force goes to *zero* – meaning the cart loses contact with the track – the cart will no longer be in circular motion. This occurs at a *critical speed*

$$v_c = \sqrt{gr}.$$

Any speed *less* than this and the cart won't complete the loop.