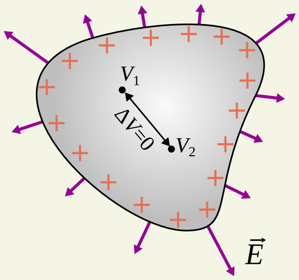
6 Batteries and Capacitance

Conductors in Electrostatic Equilibrium

In a conductor (charged or neutral) that is in *electrostatic equilibrium*, all charges are at rest. This leads to:

- All excess charge is on the surface.
- The entire conductor is all at the same potential.
- The electric field inside is zero.
- The exterior electric field is perpendicular to the surface.
- The surface charge density and electric field strength is largest at sharp corners.



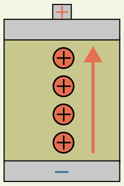
Batteries

Potential difference ΔV is caused by separation of positive and negative charge. Batteries create separation of charge by chemical reactions; the reactions do *work* on the positive charge to lift them to the positive terminal.

- The work done per charge is called the emf of the battery:

$$\mathcal{E} = W_{\text{chem}}/q.$$

- The charge separation creates a potential difference ΔV_{bat} between the terminals.
- An ideal battery has $\mathcal{E} = \Delta V_{\text{bat}}$; for real batteries, $\Delta V_{\text{bat}} < \mathcal{E}$.
- For batteries connected in *series* – positive terminal to negative terminal – the voltages add.



Energy in a Capacitor

Capacitors store an amount of energy given by

$$U_{\text{C}} = \frac{Q^2}{2C} = \frac{1}{2}C\Delta V^2.$$

This energy can be released very quickly. The energy is actually stored in the electric field, which has an energy density

$$u_E = \frac{\text{energy stored}}{\text{volume in which it's stored}} = \frac{\epsilon_0}{2}E^2.$$

Dielectrics

Inserting a dielectric – a polarized insulator – into a capacitor creates an induced electric field which reduces the electric field and voltage by an amount κ, the dielectric constant. This constant varies depending on the dielectric material. The overall effect of the dielectric is to increase the capacitance of the capacitor,

$$C = \kappa C_0,$$

where C_0 is the vacuum capacitance.

Capacitors

Two equally but oppositely charged conductors form a capacitor. The potential difference ΔV across the plates of a capacitor is proportional to the charge on each plate Q. The proportionality constant is called the *capacitance* C:

$$C = Q/\Delta V.$$

The unit of capacitance is the Farad (F): 1 F = 1 C/V. It's more useful to turn around the equation above to find the amount of charge on a capacitor:

$$Q = C\Delta V.$$

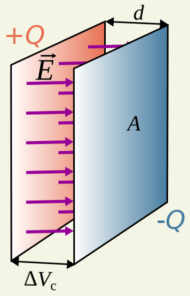
The Parallel Plate Capacitor

For a capacitor made from two parallel plates of area A and separated by distance d, the potential difference and electric field are related by

$$\Delta V_{\text{C}} = Ed.$$

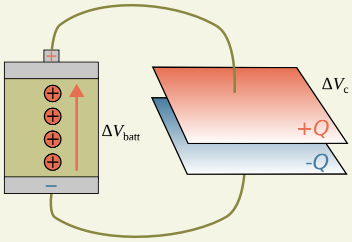
The *capacitance* is given by

$$C = \epsilon_0 A/d.$$



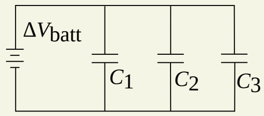
Batteries and Capacitors

A capacitor is charged by connecting it to a battery, which does work on the charge in the battery, moving it to the capacitor plate. This drives a current, in which charges flow along the wire to the capacitor. This current continues until the potential difference of the capacitor is equal to that of the battery. More than one capacitor can be connected to a battery. Two possible connections are *parallel* and *series*.



Capacitors in Parallel When connected in parallel, capacitors have the same voltage ΔV_{C} but different charge. Capacitors add to an *equivalent capacitance*:

$$C_{\text{eq}} = C_1 + C_2 + C_3 + \cdots$$



Capacitors in Series When connected in series, capacitors have the same charge Q but different voltages. Capacitors add in inverse:

$$C_{\text{eq}} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots\right)^{-1}.$$