5 Potential Energy and the Electric Potential

Mechanical Energy Conservation

Recall that in isolated, dissipationless systems. the mechanical energy is conserved:

$$K_i + U_i = K_f + U_f.$$

The potential energy depends on the type of interaction between objects.

Electric Potential Energy

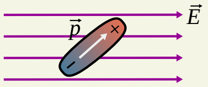
If two point charges a distance r apart interact through the electric force, they have a potential energy given by

$$U_{\text{elec}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}.$$

With three or more point charges interacting, the electric potential energy must be calculated by summing over all pairs (i, j) of particles:

$$U_{\text{elec}} = \sum_{i<j} \frac{1}{4\pi\epsilon_0}\frac{q_i q_j}{r_{ij}}.$$

When an electric dipole interacts with a constant electric field $\vec{E}$, the potential energy of the system is



$$U_{\text{elec}} = -\vec{p}\cdot\vec{E},$$

where $\vec{p}$ is the dipole moment with magnitude $p = qs$.

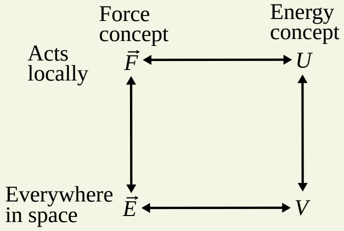
Electric Field and Potential

The electric field and the electric potential are related in the same way that force and potential energy are related. Mathematically,

$$\Delta V = V_f - V_i = -\int_{s_i}^{s_f} E_s\, ds$$

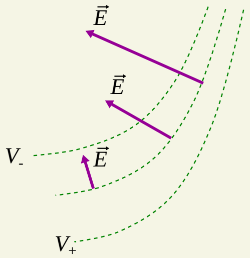
and

$$E_s = -\frac{dV}{ds}.$$



Graphically, we can see the relationship with field lines and equipotential surfaces:

- $\vec{E}$ is perpendicular to the equipotential surfaces.
- $\vec{E}$ points from higher to lower potential.
- $\vec{E}$ is greater in magnitude where the equipotential surfaces are closer together.



Electric Potential

The electric potential is another way to describe the alteration of space created by charge. It is defined through its effect on a test charge q:

$$V = \frac{U_{q+\text{ sources}}}{q}.$$

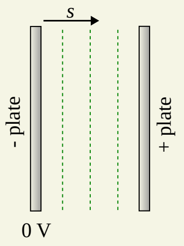
From this definition, we see that a point charge q in an electric potential V has potential energy

$$U_{\text{elec}} = qV.$$

The unit of the electric potential is the *volt*, 1 V = 1 J/C.

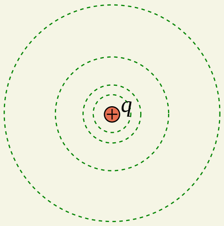
Potential inside a capacitor:

$$V = Es$$



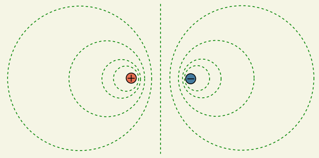
Potential of a point charge:

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$



Potential of multiple point charges:

$$V = \sum_i \frac{1}{4\pi\epsilon_0}\frac{q_i}{r_i}$$



Potential of continuous charge distributions

If the charge distribution is continuous, we again use the *principle of superposition:*

1. Divide the total charge Q of the object into many small charges dQ, each of which acts like a point charge.
2. Find the electric potential dV of each small charge dQ.
3. Add all electric potential dV together by integrating over the object to find the net potential V.

But remember: the electric potential is a *scalar*, so you don't need to break it up into x and y components.