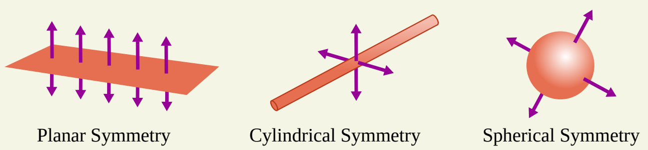
# 4 Gauss's Law

## Symmetry

A charge distribution is symmetric if it is unchanged following a translation, rotation, or reflection.

**The symmetry of the electric field must match the symmetry of the charge distribution.**

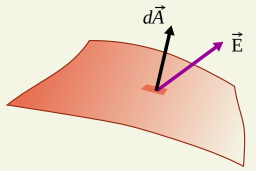
There are three main symmetries we'll encounter:




## Electric Flux

The electric flux $\Phi_E$ is, loosely speaking, how much of the electric field $\vec{E}$ "flows through" an area $A$. Mathematically,

$$\Phi_E = \int_{\text{surface}} \vec{E} \cdot d\vec{A}.$$




If the electric field is everywhere tangent to the area, $\Phi_E = 0$. If the electric field is everywhere perpendicular to the area,

$$\Phi_E = \int_{\text{surface}} E\, dA.$$

If the electric field is perpendicular and is constant in magnitude along the surface,

$$\Phi_E = EA.$$

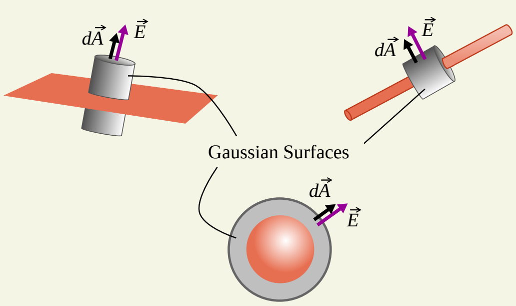
## Gauss's Law

The net electric flux through a closed surface enclosing charge $Q_{\text{in}}$ is

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\epsilon_0}.$$

## Gaussian Surfaces

When working with Gauss's Law, we draw an imaginary closed surface around charge called a *Gaussian surface*. The electric flux is calculated for this surface.




## Problem-Solving Strategy

- Determine the symmetry of the problem; if it's planar, cylindrical, or spherical, use Gauss's law.
- Draw a Gaussian surface of the same symmetry (using a cylinder or sphere). Make it as large as you need to.
- Determine the flux through the Gaussian surface; remember to use the size of the Gaussian surface, not the object itself.
- Determine the enclosed charge $Q_{\text{in}}$; depending on the situation, you might have to use the charge density and the enclosed length, area, or volume:

$$Q_{\text{in}} = \lambda L, \quad Q_{\text{in}} = \eta A, \quad \text{or} \quad Q_{\text{in}} = \rho V.$$

- Use Gauss's law to equate the flux with the enclosed charge (divided by $\epsilon_0$). Solve for what you're looking for.

## Conductors

Gauss's law allows us to make certain conclusions about *electric fields* and *conductors*.

- The Electric field is zero within a conductor.
- Any excess charge exists on the exterior surface.
- The electric field is zero within any hole in the conductor (unless there is charge in the hole).
- The electric field is perpendicular to the surface of a conductor.