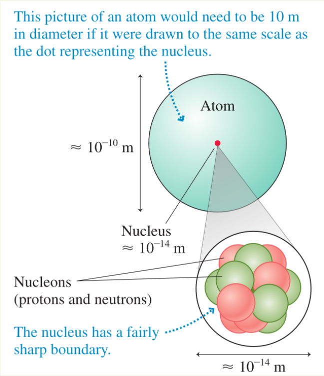
12 Nuclear Physics

The Nucleus

The nucleus of an atom consists of protons and neutrons, which are referred to as *nucleons*.

- The proton has a charge $+e$ and mass 1.00728 u $= 938.27$ MeV/c^2. The number of protons in a nucleus is Z.
- The neutron is neutral and has a mass 1.00866 u $= 939.56$ MeV/c^2. The number of neutrons in a nucleus is N.
- The mass number of a nucleus is the total number of nucleons, $A = Z + N$. Atoms with different values of N are called *isotopes*.



The radius of a nucleus with mass number A is

$$R = r_0 A^{1/3},$$

where $r_0 = 1.2$ fm. The volume of the nucleus is therefore proportional to A. This implies that nucleons are *incompressible*, tightly packed within the nucleus, and that nuclear matter has a constant density of approximately

$$\rho_{\text{nuc}} = 2.3 \times 10^{17}\ \text{kg/m}^3.$$

Mass Units

Physicists use different units for the very tiny masses in nuclear physics:

- Atomic mass: $1\ \text{u} = 1.6605 \times 10^{-27}$ kg
- Mass-energy: $1\ \text{MeV}/c^2 = 1.7827 \times 10^{-30}$ kg

So $1\ \text{u} = 931.49\ \text{MeV}/c^2$.

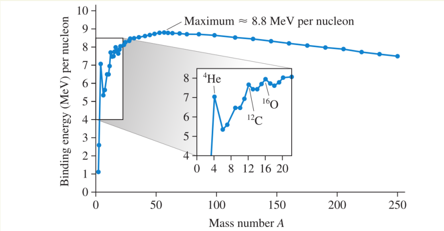
Binding Energy

A nucleus is a bound system, and the energy to disassemble the nucleus of a particular atom is called the *binding energy* B:

$$B = (Zm_{\text{H}} + Nm_{\text{n}} - m_{\text{atom}}) \times (931.49\ \text{MeV/u}),$$

where masses are in atomic mass units.

The binding energy per nucleon varies between different nuclei as shown below. The maximum is for iron; heavier nuclei than iron could become more stable by breaking into smaller nuclei, while lighter nuclei could becomes more stable by fusing together into larger nuclei.



Nuclear Forces

Interactions within the nucleus are governed by two nuclear forces:

- The *strong nuclear force*, which is an attractive force between any two nucleons. It is a short-range force and is stronger than the electromagnetic force.
- The *weak nuclear force* is involved in the process of beta decay, which turns a neutron into a proton, emitting an electron and antineutrino, as well as the reverse reaction.

Radioactivity

Many nuclei are unstable and decay to a different nucleus while emitting an alpha (α), beta (β), or gamma (γ) ray. This is a random quantum mechanical process. The *decay rate* r is the probability that a particular nuclei will decay within the next second. The number of nuclei N in a sample will undergo exponential decay:

$$N = N_0 e^{-t/\tau},$$

where N_0 is the number of nuclei at time $t = 0$ and $\tau = 1/r$ is the *lifetime* of the nucleus.

More commonly is the half-life $t_{1/2}$, which is the time in which half of a sample of radioactive atoms decays:

$$N = N_0 \left(\frac{1}{2}\right)^{t/t_{1/2}}.$$

The lifetime and half-life are related by $t_{1/2} = \tau \ln 2$.

The *activity* R of a radioactive sample is the number of decays per second:

$$R = \left|\frac{dN}{dt}\right| = R_0 \left(\frac{1}{2}\right)^{t/t_{1/2}},$$

where $R_0 = rN_0$ is the activity at $t = 0$. The SI unit of activity is the becquerel (1 Bq = 1 decay/s), although curies are also popular:

$$1\ \text{Ci} = 3.7 \times 10^{10}\ \text{Bq}.$$

Alpha Decay

A heavy parent nucleus X can decay to a lighter daughter nucleus Y by emitting an alpha particle (a ^{4}He nucleus):

$$^A\text{X}_Z \rightarrow {}^{A-4}\text{Y}_{Z-2} + \alpha + \text{energy}.$$

The energy goes almost entirely into the kinetic energy of the alpha particle:

$$K_\alpha = (m_{\text{X}} - m_{\text{Y}} - m_{\text{He}})c^2.$$

Beta Decay

Nuclei with too many protons or neutrons can undergo beta decay to move closer to the *line of stability*. The decay can emit an electron (beta-minus decay) or a positron (beta-plus decay):

$$^A\text{X}_Z \rightarrow {}^A\text{Y}_{Z+1} + \text{e}^- + \text{energy}, \quad {}^A\text{X}_Z \rightarrow {}^A\text{Y}_{Z-1} + \text{e}^+ + \text{energy}$$

Gamma Decay

Gamma decay occurs when a nucleus makes a quantum transition from an excited energy state to a lower-energy state by emitting a high-energy photon. Alpha and beta decay often leave the daughter nucleus in an excited state, so gamma emission is usually found to accompany those decays.