Compound interest

The compound interest formula

When interest is **compounded**, the interest earned in each period is added to the principal, so future interest is calculated on a larger amount. This produces exponential growth.

$$\boxed{FV = PV\left(1 + \frac{r}{m}\right)^{mt}}$$

FV	Future value (total amount at end of investment)
PV	Present value (principal invested or borrowed)
r	Nominal annual interest rate (as a decimal)
m	Number of compounding periods per year
t	Duration of investment in years

Common compounding periods:

Frequency	Annually	Semi-annually	Quarterly	Monthly	Weekly	Daily
m	1	2	4	12	52	365

Note: Always convert r to a decimal before substituting. For example, $4.5\% = 0.045$.

Solving for future value

Example: \$3,200 is invested at 4.5% compounded quarterly for 6 years. Find the future value and the interest earned.

Solution: Here $PV = 3200$, $r = 0.045$, $m = 4$, $t = 6$.

$$FV = 3200\left(1 + \frac{0.045}{4}\right)^{4\times 6} = 3200(1.01125)^{24}$$

$$FV = 3200(1.30862\ldots) \approx \$4{,}187.57$$

Interest earned $= FV - PV = 4187.68 - 3200 = \$987.57$.

Solving for present value

Rearrange the formula to isolate PV:

$$PV = FV\left(1 + \frac{r}{m}\right)^{-mt}$$

Student Learning Support, Teaching and Learning Centre
studentlearning@ontariotechu.ca
ontariotechu.ca/studentlearning

OntarioTech UNIVERSITY

Example: How much must you invest today at 5% compounded monthly to have $10,000 in 8 years?

Solution: Here $FV = 10000$, $r = 0.05$, $m = 12$, $t = 8$.

$$PV = 10000\left(1 + \frac{0.05}{12}\right)^{-12\times 8} = 10000(1.00\overline{416})^{-96}$$

$$PV = 10000(0.67121\ldots) \approx \$6{,}708.77$$

Solving for the interest rate

Isolate the base, then raise both sides to the power $\frac{1}{mt}$:

$$\left(1 + \frac{r}{m}\right) = \left(\frac{FV}{PV}\right)^{\frac{1}{mt}} \implies r = m\left[\left(\frac{FV}{PV}\right)^{\frac{1}{mt}} - 1\right]$$

Example: An investment of $4,500 grew to $6,100 over 7 years compounded semi-annually. Find the nominal annual interest rate.

Solution: Here $PV = 4500$, $FV = 6100$, $m = 2$, $t = 7$.

$$1 + \frac{r}{2} = \left(\frac{6100}{4500}\right)^{\frac{1}{14}} = (1.35\overline{5})^{0.07143} = 1.02193\ldots$$

$$r = 2(1.02193\ldots - 1) = 2(0.02193) \approx 0.0439$$

The nominal annual rate is approximately **4.39%**.

Solving for time

When the unknown is in the exponent, take the **natural logarithm** of both sides and apply $\ln(x^y) = y\ln(x)$. *See the logarithms tipsheet for a review of log rules.*

$$FV = PV\left(1 + \frac{r}{m}\right)^{mt} \implies \ln\left(\frac{FV}{PV}\right) = mt \cdot \ln\left(1 + \frac{r}{m}\right) \implies t = \frac{\ln(FV/PV)}{m \cdot \ln(1 + r/m)}$$

Example: How long will it take $1,000 to grow to $1,500 at 6% compounded monthly?

Solution: Here $PV = 1000$, $FV = 1500$, $r = 0.06$, $m = 12$.

$$t = \frac{\ln(1500/1000)}{12 \cdot \ln(1 + 0.06/12)} = \frac{\ln(1.5)}{12 \cdot \ln(1.005)} = \frac{0.40546}{12(0.004987)} = \frac{0.40546}{0.05985} \approx 6.77 \text{ years}$$

Example: At 8% compounded semi-annually, how long does it take money to double?

Student Learning Support, Teaching and Learning Centre
studentlearning@ontariotechu.ca
ontariotechu.ca/studentlearning

Solution: Set $FV = 2 \cdot PV$ (the PV cancels):

$$t = \frac{\ln(2)}{2 \cdot \ln(1.04)} = \frac{0.69315}{2(0.03922)} = \frac{0.69315}{0.07844} \approx 8.84 \text{ years}$$

Summary: isolating each variable

Unknown	Rearranged formula
FV	$FV = PV\left(1 + \frac{r}{m}\right)^{mt}$
PV	$PV = FV\left(1 + \frac{r}{m}\right)^{-mt}$
r	$r = m\left[\left(\frac{FV}{PV}\right)^{1/(mt)} - 1\right]$
t	$t = \frac{\ln(FV/PV)}{m \cdot \ln(1 + r/m)}$

Caution: Duration t must always be in **years**. Convert months to years before substituting: e.g., 9 months $= 0.75$ years, 18 months $= 1.5$ years.

Practice problems. Solve each problem. Round dollar amounts to the nearest cent and rates to two decimal places.

1. Find the future value of \$2,500 invested at 3.6% compounded monthly for 4 years.

2. Find the future value of \$800 invested at 5% compounded semi-annually for 10 years.

3. How much must you invest today at 4% compounded quarterly to have \$5,000 in 6 years?

4. How much must you invest today at 6.5% compounded monthly to have \$12,000 in 8 years?

5. \$1,200 grew to \$1,550 in 5 years compounded quarterly. Find the nominal annual rate.

6. \$3,000 grew to \$4,800 in 9 years compounded monthly. Find the nominal annual rate.

7. How long will it take \$600 to grow to \$900 at 5% compounded semi-annually?

8. How long will it take money to double at 6% compounded monthly?

9. Find the interest earned on \$4,000 invested at 3.2% compounded weekly for 7 years.

10. You need \$20,000 in 10 years. If you can earn 5.5% compounded semi-annually, how much do you invest today?

Answers: 1. \$2,886.59 2. \$1,310.89 3. \$3,937.83 4. \$7,144.26 5. 5.15% 6. 5.23% 7. 8.21 years 8. 11.58 years 9. \$1,003.94 10. \$11,625.01

Student Learning Support, Teaching and Learning Centre
studentlearning@ontariotechu.ca
ontariotechu.ca/studentlearning