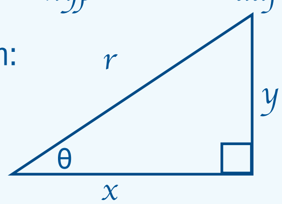
Trigonometry

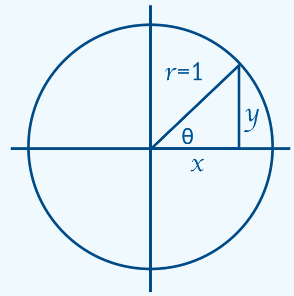
Right Angle Triangle Definition (for $0 < \theta < \frac{\pi}{2}$):

$\sin(\theta) = \frac{opp}{hyp}$ $\cos(\theta) = \frac{adj}{hyp}$ $\tan(\theta) = \frac{opp}{adj}$

Pythagorean Theorem:
$r^2 = x^2 + y^2$



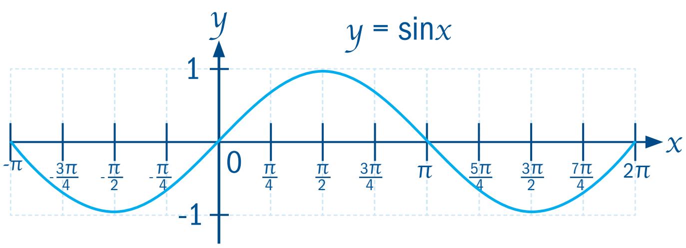
The Unit Circle Definition (for $\theta \in \mathcal{R}$):



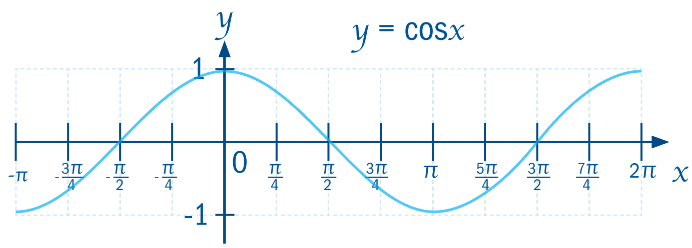
$\sin(\theta) = \frac{y}{1} = y$

$\cos(\theta) = \frac{x}{1} = x$

$\tan(\theta) = \frac{x}{y}$

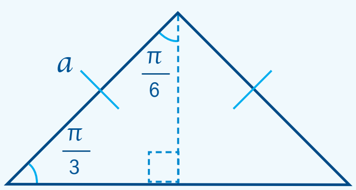


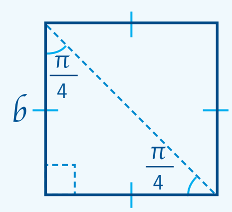
$\sin(x)=0$ for $x = k\pi, k \in Z$



$\cos(x)=0$ for $x = \frac{\pi}{2} + k\pi, k \in Z$

Special triangles are simply right angle triangles found within equilateral triangles and squares:





For simplicity, and so the side lengths are not fractions, set the sides of the equilateral triangle to a=2 and the square to b=1.

Inverse Trigonometric Functions:

$y = \sin^{-1}(x) = \arcsin(x)$ where $-1 \leq x \leq 1$ and $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

$y = \cos^{-1}(x) = \arccos(x)$ where $-1 \leq x \leq 1$ and $0 \leq y \leq \pi$

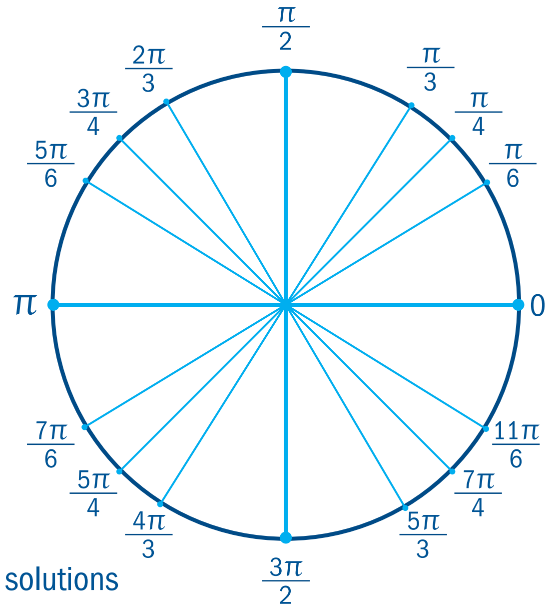
$y = \tan^{-1}(x) = \arctan(x)$ where $-\infty < x < \infty$ and $0 < y < \frac{\pi}{2}$

NOTE: Inverse functions are not reciprocal functions, i.e. $\sin^{-1}(x) \neq \frac{1}{\sin(x)}$. This is a common misconception since we use the notation like $\sin^2(x) = (\sin(x))^2$ to make other exponents easier to write. The exception is that we use $f^{-1}(x)$ to indicate the inverse of $f(x)$.

Reciprocal Trigonometric Functions:

$\csc(\theta) = \frac{1}{\sin(\theta)}$ $\sec(\theta) = \frac{1}{\cos(\theta)}$ $\cot(\theta) = \frac{1}{\tan(\theta)}$

Why Radians?



Degrees are not based on the fundamental properties of a circle, so they are awkward to use in advanced math. Instead, we use radians, which divide up the circle based on circumference.

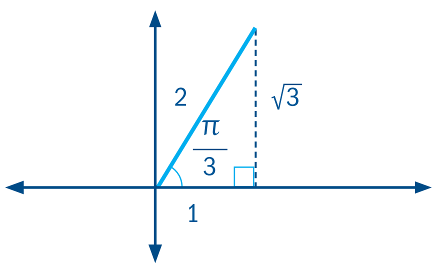
A full turn in a circle is $2\pi r$. So for angles we use fractions of 2π radians (a full turn).

Solving Trigonometric Equations:

When solving an equation such as $\sin(\theta)=\frac{\sqrt{3}}{2}$, there are infinite solutions (since the function is periodic), unless a restricted domain is given. This is different from the equation $\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$, which has only one solution since an inverse function has only one output for each input.

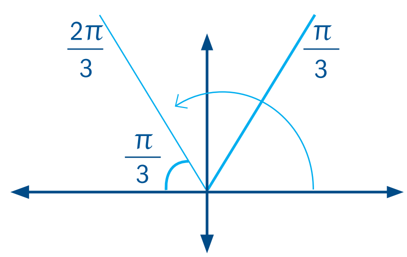
Find all solutions of $\sin(\theta)=\frac{\sqrt{3}}{2}$ on the interval $0 \le \theta \le 2\pi$:

First, find the acute angle from the +x axis. This ratio can be recognized as belonging to a special triangle.



Recall:
$\sin(\theta)=\frac{opp}{hyp}$. The angle with an opposite side of $\sqrt{3}$ and a hypotenuse of 2 is $\frac{\pi}{3}$.

Next, find the other angles in the given interval that will have the same trigonometric ratio. Recall that $\sin(\theta)=\frac{y}{r}$ so the next angle is in the other quadrant where y is positive. So this second angle, measured from the +x axis, is $\pi-\frac{\pi}{3}=\frac{2\pi}{3}$.



Therefore, the solutions to $\sin(\theta)=\frac{\sqrt{3}}{2}$ on the interval $0 \le \theta \le 2\pi$ are $\theta=\frac{\pi}{3},\frac{2\pi}{3}$.

Extend: Confirm that the solutions on the interval $2\pi \le \theta \le 4\pi$ would be $\theta=\frac{7\pi}{3},\frac{8\pi}{3}$.

You Try: Find all the solutions to $\sqrt{2}\cos(\theta)+1=0$ on the interval $0 \le \theta \le 2\pi$. (Answer: $\theta=\frac{3\pi}{4},\frac{5\pi}{4}$)

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