Gauss's Law

Conductors	Insulators
• Charge is distributed uniformly on the surface or surfaces ($\boldsymbol{\eta}$ is used). • Polarization occurs.	• Charge is distributed uniformly throughout the area if the object is 2D ($\boldsymbol{\eta}$ is used) or the volume if the object is 3D ($\boldsymbol{\rho}$ is used).

Conductor example

A conducting solid sphere and a spherical shell are concentric as shown in the figure below. The inner sphere carries a charge of +5.00 μC. The outer conducting shell carries a net charge of -8.00 μC. The radii shown in the figure have the values R_1 = 10.0 cm, R_2 = 20.0 cm, and R_3 = 30.0 cm.

i) Find the total excess charge on the inner and outer surfaces of the conducting sphere.

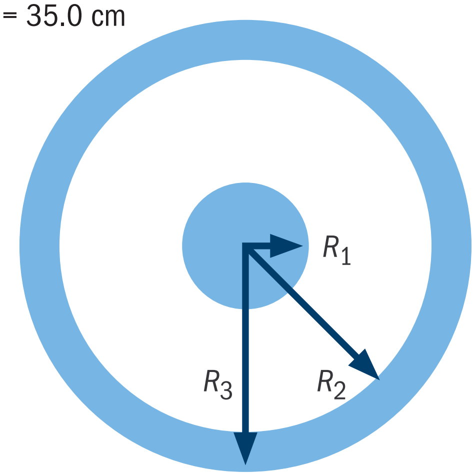
ii) Find the magnitude and direction of the electric at the following distances r from the centre of the inner sphere.

iii) r = 9.5 cm

iv) r = 15.0 cm

v) r = 27.0 cm

vi) r = 35.0 cm



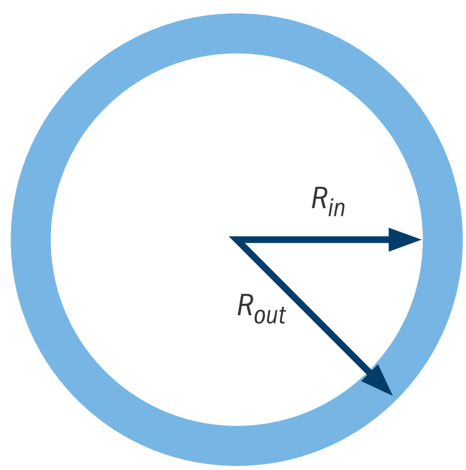
Insulator example

A spherical, non-conducting shell of inner radius r_{in} = 10.0 cm and outer radius r_{out} = 15.0 cm carries a total charge Q = 15.0 μC distributed uniformly throughout the volume of the shell. What is the magnitude of the electric field from the center of the shell at a distance:

i) r = 8.0 cm

ii) r = 12.0 cm

iii) r = 20.0 cm



Check the next page for solutions.

Solution	**Solution**
i) $q_{shell} = q_{inner} + q_{outer}$ $-8\mu C = -5\mu\, C + q_{outer}$ $q_{outer} = -3\mu C$ ii) $E = \frac{Q_{enc}}{A\,\epsilon_0}$ iii) r = 0.095 m $r < 0.1\, m = Q_{enc} = 0 \;\; \rightarrow E = 0$ iv) r = 0.15 m $E = \dfrac{Q_{enc}}{A\,\epsilon_0}$ $A = 4\pi r^2$ $(surface\ area\ of\ the\ imaginary\ sphere)$ $\epsilon_0 = (8.85)10^{-12}$ $E = \dfrac{(5)\,10^{-6}}{4\pi\,(0.15)^2\,(8.85)\,10^{-12}}$ $E = (2)\,10^6\,N/C\ (radially\ outwards)$ v) r = 0.27 m $Q_{enc} = 5\mu C - 5\mu C = 0 \;\; \rightarrow E = 0\,N/C$ vi) r = 0.35 m $E = \dfrac{Q_{enc}}{A\,\epsilon_0} = \dfrac{(-3)\,10^{-6}}{4\pi\,(0.35)^2\,(8.85)\,10^{-12}} \rightarrow$ $E = (-2.2)\,10^5\,N/C\ \ (radially\ inwards)$	i) r = 8.0 cm $r < 0.1\,m = Q_{enc} = 0 \;\; \rightarrow E = 0$ ii) r = 12.0 cm $\rho = \dfrac{Q}{V} = \dfrac{(15)\,10^{-6}}{\frac{4}{3}\pi\,(0.15^3 - 0.1^3)} = 0.001508\,C/m^3$ $Q_{enclosed} = \rho\,V_{gauss} = \rho\left[\frac{4}{3}\,\pi\,(r^3 - r_{in}{}^3)\right]$ $Q_{enclosed} = 0.001508\left[\frac{4}{3}\,\pi\,(0.12^3 - 0.1^3)\right]$ $= (4.5986)\,10^{-6}\,C$ $E = \dfrac{Q_{gauss}}{A_{gauss}\,\epsilon_0} = \dfrac{(4.5986)\,10^{-6}}{4\pi\,(0.12)^2\,(8.85)\,10^{-12}}$ $= (2.87)\,10^6\,N/C$ iii) r = 20.0 cm $r > 0.15\,m = Q_{enc} = 15\mu C$ $E = \dfrac{Q_{gauss}}{A_{gauss}\,\epsilon_0} = \dfrac{(15)10^{-6}}{4\pi\,(0.2)^2\,(8.85)\,10^{-12}} =$ $E = (3.372)10^6\,N/C$