# Electric Field Rod Integrals

Steps for solving electric field integral problems.

1. Show an arbitrary segment. Assume the length of the rod of $L$, charge of the rod of $Q$, charge of the segment of $dq$, linear charge density of $\lambda$.

Horizontal rod.

$dx$
the length or width
of the segment.

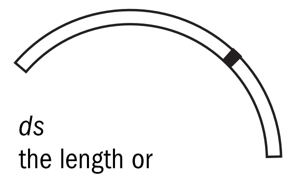
Keep all the terms as a function of $x$ since $x$ is the variable.

Vertical rod.

$dy$
the length or height
of the segment.

Keep all the terms as a function of $y$ since $y$ is the variable.

Circular rod.



$ds$
the length or
arclength
of the segment.

Keep all the terms as a function of $\theta$ since $\theta$ is the variable.

2. Isolate for $dq$ from the linear charge density relations:

$$\lambda = \frac{Q}{L} = \frac{dq}{dx}$$
$$dq = \frac{Q}{L} dx$$

$$\lambda = \frac{Q}{L} = \frac{dq}{dy}$$
$$dq = \frac{Q}{L} dy$$

$$\lambda = \frac{Q}{L} = \frac{dq}{ds}$$
$$ds = R\, d\theta$$
$$dq = \frac{Q}{L} R\, d\theta$$

3. Rewrite $dE$ equation by plugging in $dq$ from the previous step:

$$dE = k\frac{dq}{r^2} = \frac{kQ}{L}\frac{dx}{r^2}$$

$$dE = k\frac{dq}{r^2} = \frac{kQ}{L}\frac{dy}{r^2}$$

$$dE = k\frac{dq}{r^2} = \frac{kQ}{L}\frac{R\, d\theta}{r^2}$$

4. Show $d\vec{E}$ of the segment on the given point and write it in component form:

$d\vec{E} = dE_x\hat{\imath} + dE_y\hat{\jmath}$ or $d\vec{E} = dE\,(cos\theta\hat{\imath} + sin\theta\hat{\jmath})$

**Tip:** If the rod is symmetrical about $x$ or $y$, there might be a cancellation. Either $x$-components or $y$-components of the sum of $d\vec{E}$ vectors cancel out. $\vec{E} = \int d\vec{E}$

5. Check for bounds when solving the integral.

These are the marginal values of the variables:

$x$ values of the left and right edges of the rod.

$y$ values of the top and bottom edges of the rod.

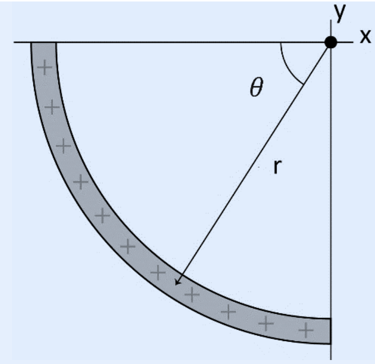
$\theta$ values for the marginal segments.

6. Solve for the integral.

Check the formula sheet for the given equations.

# Example one:

2.80 nC of charge is uniformly distributed along a thin rod of length $L$ = 6.4 cm, which is then bent into a quarter circle as shown in the figure below. What is the magnitude and the direction of the electric field at the centre of the circle?




$$\lambda = \frac{Q}{L} = \frac{dq}{ds} \qquad ds = rd\theta \rightarrow \frac{Q}{L} = \frac{dq}{rd\theta}$$

$$dq = \frac{Q}{L} rd\theta \qquad L = \frac{2\pi r}{4} \rightarrow r = \frac{2L}{\pi}$$

$$dE = \frac{kdq}{r^2} = \frac{k}{r^2}\frac{Q}{L} rd\theta \rightarrow \frac{kQ}{Lr}\,d\theta$$

$$dE = \frac{kQ}{L\,\frac{2L}{\pi}}\,d\theta \rightarrow dE = \frac{kQ\pi}{2L^2} d\theta$$

$$d\vec{E} = dEcos\theta\,\hat{\imath} + dEsin\theta\,\hat{\jmath}$$

$$\vec{E} = \int d\vec{E} = \frac{kQ\pi}{2L^2}\int_0^{\frac{\pi}{2}} cos\theta\,\hat{\imath} d\theta + \frac{kQ\pi}{2L^2}\int_0^{\frac{\pi}{2}} sin\theta\,\hat{\jmath}\,d\theta$$

$$\vec{E} = \frac{kQ\pi}{2L^2}[sin\theta]\hat{\imath} + \frac{kQ\pi}{2L^2}[-cos\theta]\,\hat{\jmath} = \frac{kQ\pi}{2L^2}[1-0]\hat{\imath} - \frac{kQ\pi}{2L^2}[0-1]\,\hat{\jmath}$$

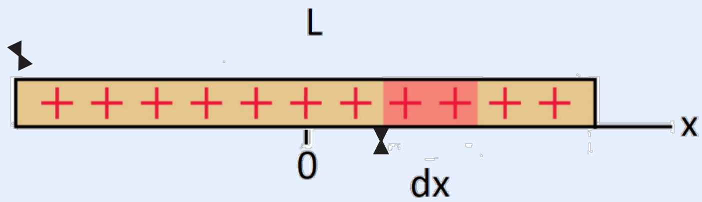
$$\vec{E} = \frac{kQ\pi}{2L^2}\hat{\imath} + \frac{kQ\pi}{2L^2}\,\hat{\jmath}$$

$$E = \sqrt{E_X^2 + E_y^2} = \sqrt{\left(\frac{kQ\pi}{2L^2}\right)^2 + \left(\frac{kQ\pi}{2L^2}\right)^2} = \sqrt{2\left(\frac{kQ\pi}{2L^2}\right)^2}$$

$$E = \sqrt{2\left(\frac{(8.99)10^9\,(2.80)10^{-9}\,\pi}{2((6.40)\,10^{-2})^2}\right)^2} = 13652\ N/C$$

# Example two:

2.80nC of charge is uniformly distributed along a thin rod of length *L* = 6.4 cm. What is the magnitude and the direction of the electric field at point *P*, a distance *r* = 8.8 cm from the centre of the rod?




$$\lambda = \frac{Q}{L} = \frac{dq}{dx} \rightarrow dq = \frac{Q}{L}dx$$

$$dE = \frac{kdq}{(r-x)^2} = \frac{kQ}{L}\frac{dx}{(r-x)^2}$$

$$E = \int dE = \int \frac{kQ}{L}\frac{dx}{(r-x)^2} = \frac{kQ}{L}\int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{dx}{(r-x)^2}$$

$$E = \frac{kQ}{L}\left[\frac{1}{(r-x)}\right]$$

$$E = \frac{kQ}{L}\left[\frac{1}{r-\frac{L}{2}} - \frac{1}{r-\left(-\frac{L}{2}\right)}\right]$$

$$E = \frac{kQ}{L}\left[\frac{1}{r-\frac{L}{2}} - \frac{1}{r+\frac{L}{2}}\right] = \frac{kQ}{L}\left[\frac{\left(r+\frac{L}{2}\right)-\left(r-\frac{L}{2}\right)}{r^2-\frac{L^2}{4}}\right] = \frac{kQ}{L}\frac{L}{r^2-\frac{L^2}{4}}$$

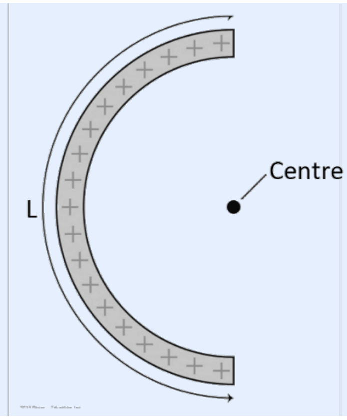
$$E = \frac{kQ}{r^2-\frac{L^2}{4}}$$

$$E = \frac{(8.99*10^9)*(2.80*10^{-9})}{0.088^2-\frac{0.064^2}{4}} = 3746\ N/C$$

$$\vec{E} = \{3750\ \hat{\imath}\}\ N/C$$

## Example three:

2.80nC of charge is uniformly distributed along a thin rod of length
*L* = 6.4 cm, which is then bent into a semi-circle as shown in the figure. What is the magnitude of the direction of the electric field at the centre of the circle?




$$\lambda = \frac{Q}{L} = \frac{dq}{ds}, \qquad ds = rd\theta \ \rightarrow \frac{Q}{L} = \frac{dq}{rd\theta}$$

$$dq = \frac{Q}{L} rd\theta \qquad L = \frac{2\pi r}{2} \ \rightarrow r = \frac{L}{\pi}$$

$$d\vec{E} = \frac{kdq}{r^2} = \frac{k}{r^2}\frac{Q}{L} rd\theta \ \rightarrow \frac{kQ}{Lr}\ d\theta$$

$$\vec{E} = \int \quad dEcos\theta\ \hat{\imath} = \frac{kQ}{Lr}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \quad cos\theta\ d\theta\ \hat{\imath}$$

$$\vec{E} = \frac{kQ}{Lr} sin\theta\ \hat{\imath}$$

$$\vec{E} = \frac{kQ}{Lr}\left[sin\frac{\pi}{2} - sin\ sin\ \left(-\frac{\pi}{2}\right)\right]\ \hat{\imath} = \frac{kQ}{Lr}\left(1 - (-1)\right)\hat{\imath} = \frac{kQ}{Lr}(2)\hat{\imath}$$

$$\vec{E} = \frac{2kQ}{L\,\frac{L}{\pi}}\hat{\imath} = \frac{2kQ\pi}{L^2}\ \hat{\imath}$$

$$\vec{E} = \frac{2kQ\pi}{L^2}\ \hat{\imath} = \frac{2\ (8.99)\ 10^9\ (2.8)10^{-9}\ \pi}{0.064^2}\ \hat{\imath}$$

$$\vec{E} = \{(3.86)\ 10^4\ \hat{\imath}\}\ N/C$$

**Student Learning Centre**

**Call:** 905.721.8668 ext. 6578
**Email:** studentlearning@ontariotechu.ca
**Website:** ontariotechu.ca/studentlearning
**Downtown Oshawa campus location:** Charles Hall
**North Oshawa campus location:** Shawenjigewining Hall