Formulas for pre-calculus concepts

ARITHMETIC OPERATIONS

$a(b+c)=ab+ac$ $\quad \frac{a+c}{b}=\frac{a}{b}+\frac{c}{b}$ $\quad \frac{a/b}{c/d}=\frac{a}{b}\times\frac{d}{c}=\frac{ad}{bc}$ $\quad \frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$

FACTORING FORMULAS

$a^2-b^2=(a+b)(a-b)$ $\quad a^3+b^3=(a+b)(a^2-ab+b^2)$ $\quad a^3-b^3=(a-b)(a^2+ab+b^2)$

ABSOLUTE VALUE

$|a|\geq 0$ $\quad |a|=\begin{cases} a & \text{if } a\geq 0 \\ -a & \text{if } a<0 \end{cases}$ $\quad |x|<b \longleftrightarrow -b<x<b$ $\quad |x|>b \longleftrightarrow x<-b \text{ or } x>b$

EXPONENTS AND RADICALS

$a^m\cdot a^n=a^{(m+n)}$

$\frac{a^m}{a^n}=a^{m-n}$

$(a^m)^n=a^{mn}$

$(ab)^m=a^m b^m$

$\left(\frac{a}{b}\right)^m=\frac{a^m}{b^m}$

$a^{-n}=\frac{1}{a^n}$

If n is even, $\sqrt[n]{a^n}=|a|$

If n is odd, $\sqrt[n]{a^n}=a$

$\sqrt[n]{a^m}=(\sqrt[n]{a})^m=a^{m/n}$

$\sqrt[n]{ab}=\sqrt[n]{a}\ \sqrt[n]{b}$

$\sqrt[n]{\frac{a}{b}}=\frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

TRANSFORMATION OF BASIC GRAPHS

$y=f(x)\pm b$ Vertical translation b units in the same direction as the sign

$y=f(x\pm d)$ Horizontal translation d units in the opposite direction as the sign

$y=-f(x)$ Vertical reflection across the x-axis

$y=f(-x)$ Horizontal reflection across the y-axis

$y=af(x)$ Vertical stretch by a factor of a where $a>1$

$y=\frac{1}{a}f(x)$ Vertical compression by a factor of a where $a>1$

$y=f(cx)$ Horizontal compression by a factor of c where $c>1$

$y=f(\frac{1}{c}x)$ Horizontal stretch by a factor of c where $c>1$

SYMMETRY

Even Function: $f(-x) = f(x)$

Odd Function: $f(-x) = -f(x)$

EQUATION-SOLVING PRINCIPLE

$ab = 0 \longleftrightarrow a = 0$ or $b = 0$

THE QUADRATIC FORMULA

If $ax^2+bx+c=0$ then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

PROPERTIES OF EXPONENTIAL AND LOGARITHMIC FUNCTIONS

$\log_a x = y \longleftrightarrow x = a^y$

$\ln x = y \longleftrightarrow x = e^y$

$\log_a xy = \log_a x + \log_a y$

$\log_a \dfrac{x}{y} = \log_a x - \log_a y$

$\log_e x = \ln x$

$\log_a x^b = b \log_a x$

$\log_a b = \dfrac{\log_c b}{\log_c a} = \dfrac{\ln b}{\ln a}$

$\log_a a^x = x$

$a^{\log_a x} = x$

$\ln e^x = x$

$e^{\ln x} = x$

FORMULAS INVOLVING LINES

The slope of the line containing points (x_1, y_1) and (x_2, y_2) is given by $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Slope-intercept equation with slope m and y - intercept b: $y = mx + b$

Point-slope equation: $y - y_1 = m(x - x_1)$

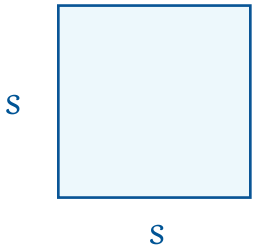
GEOMETRIC FORMULAS

Rectangle

Area: $A = lw$

Perimeter: $P = 2l + 2w$

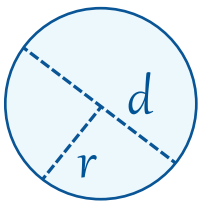
Square



Area: $A = s^2$

Perimeter: $P = 4s$

Circle



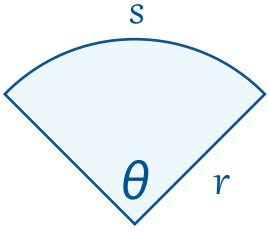
Area: $A = \pi r^2$

Circumference: $C = \pi d = 2\pi r$

Equation of the circle with centre (h,k) and radius r:

$(x - h)^2 + (y - k)^2 = r^2$

Sector of Circle



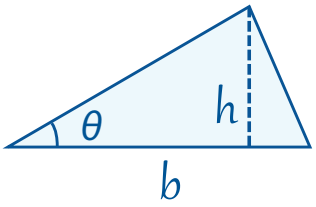
Area: $A = \dfrac{1}{2} r^2\theta$

$s = r\theta$ where θ is in radians

Triangle

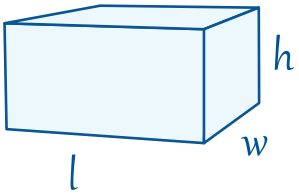
Area: $A = \frac{1}{2}bh$

$= \frac{1}{2}ab\sin\theta$



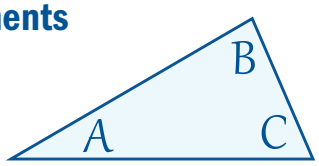
Rectangular Solid

Volume $V = lwh$



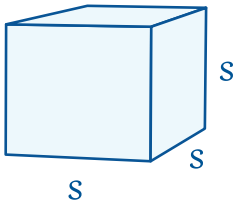
Sum of Angle Measurements

$A + B + C = 180°$



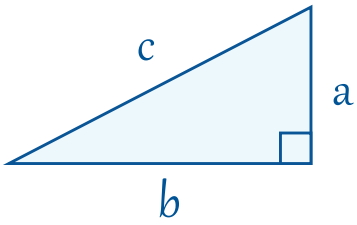
Cube

Volume $V = s^3$



Pythagorean Theorem

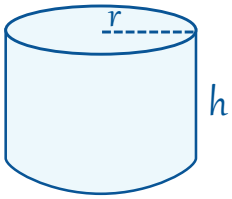
$a^2 + b^2 = c^2$



Cylinder

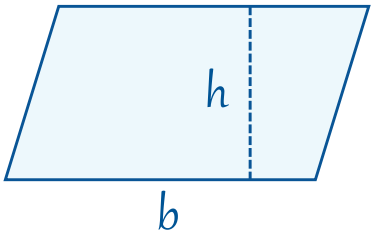
Volume: $V = \pi r^2 h$

Total surface area $S = 2\pi rh + 2\pi r^2$



Parallelogram

Area: $A = bh$

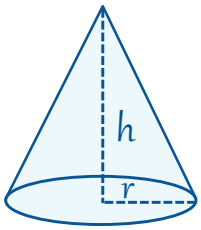


Cone

Volume: $V = \frac{1}{3}\pi r^2 h$

Total surface area:

$S = \pi r^2 + \pi r\sqrt{(r^2 + h^2)}$



DISTANCE AND MIDPOINT FORMULAS

Distance from (x_1, y_1) to (x_2, y_2):

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint of the line segment from

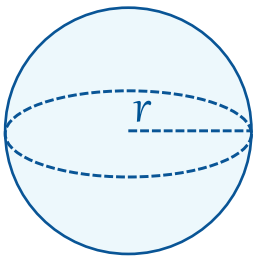
(x_1, y_1) to (x_2, y_2):

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Sphere

Volume: $V = \frac{4}{3}\pi r^3$

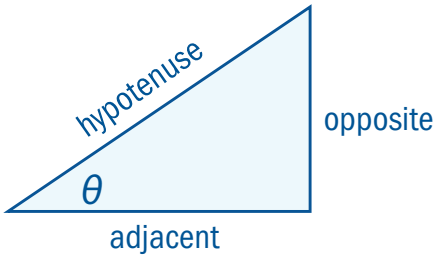
Surface area: $S = 4\pi r^2$



TRIGONOMETRY

Angle Measurement

π radians $= 180°$



Right angle Trigonometry

$$\sin\theta = \frac{opp}{hyp} \qquad \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{opp}{adj} \qquad \sec\theta = \frac{1}{\cos\theta} = \frac{hyp}{adj}$$

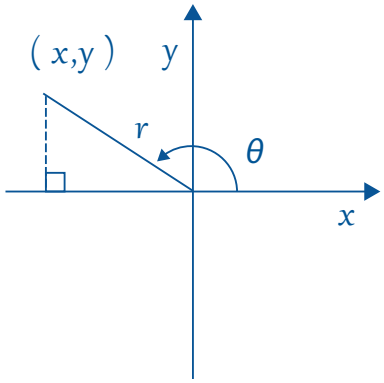
$$\cos\theta = \frac{adj}{hyp} \qquad \csc\theta = \frac{1}{\sin\theta} = \frac{hyp}{opp} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta} = \frac{adj}{opp}$$

Trigonometric Functions

$$\sin\theta = \frac{y}{r} = \frac{1}{\csc\theta}$$

$$\cos\theta = \frac{x}{r} = \frac{1}{\sec\theta}$$

$$\tan\theta = \frac{y}{x} = \frac{1}{\cot\theta}$$



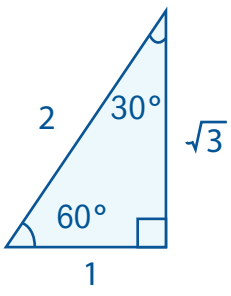
θ	Radian	Sin(θ)	cos(θ)	tan(θ)
0°	0	0	1	0
30°	π/6	1/2	$\sqrt{3}/2$	$\sqrt{3}/3$
45°	π/4	$\sqrt{2}/2$	$\sqrt{2}/2$	1
60°	π/3	$\sqrt{3}/2$	1/2	$\sqrt{3}$
90°	π/2	1	0	-

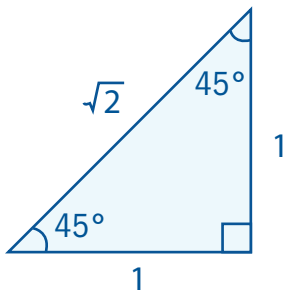
Identities

$$\sin^2 x + \cos^2 x = 1 \qquad 1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x \qquad \sin 2x = 2 \sin x \cos x$$

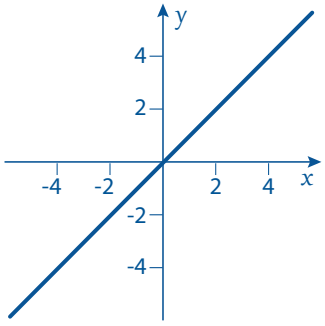
$$\cos^2 x = \frac{1}{2}(1 + \cos 2x) \qquad \sin^2 x = \frac{1}{2}(1 - \cos 2x)$$



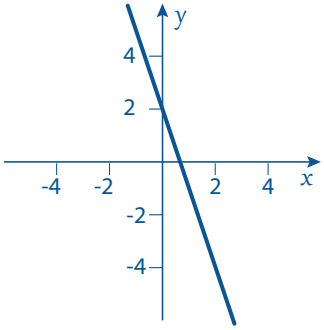


LIBRARY OF FUNCTIONS

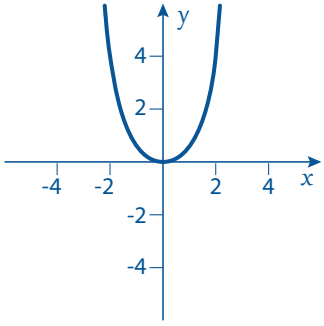
Linear Function
$f(x) = x$



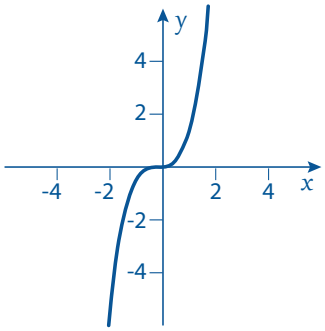
Linear Function
$f(x) = -3x + 2$



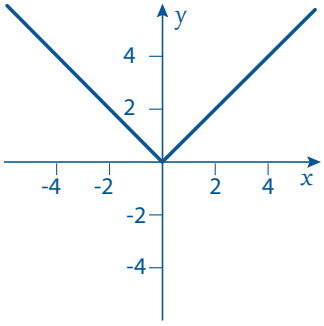
Quadratic Function
$f(x) = x^2$



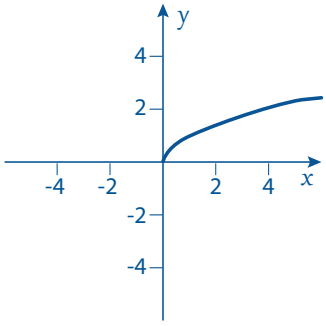
Cubic Function
$f(x) = x^3$



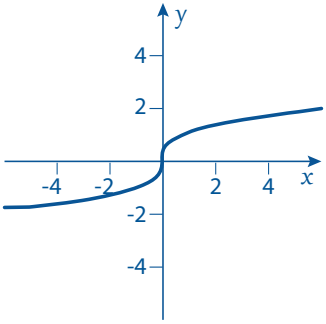
Absolute-value Function
$f(x) = |x|$



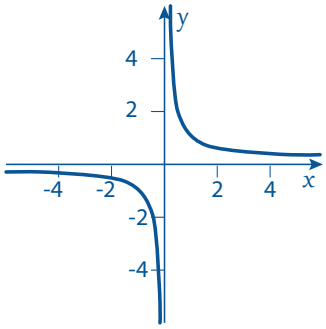
Square-root Function
$f(x) = \sqrt{x}$



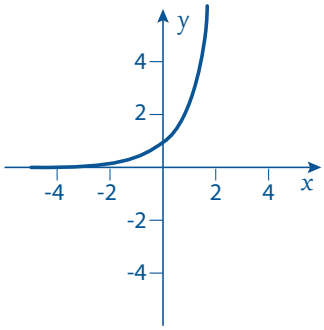
Cube-root Function
$f(x) = \sqrt[3]{x}$



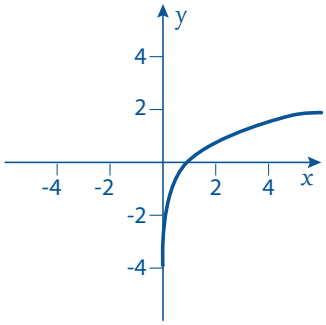
Rational Function
$f(x) = \frac{1}{x}$



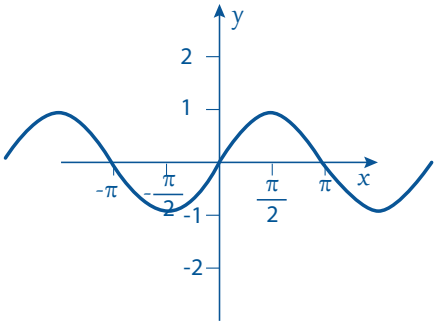
Exponential Function
$f(x) = e^x$



Logarithmic Function
$f(x) = \ln(x)$



Sine Function
$f(x) = \sin(x)$



Cosine Function
$f(x) = \cos(x)$