# Logic Operators

If p and q are propositions. A proposition is a declarative statement that states a fact that is either true or false.

## Precedence

PRECEDENCE	OPERATOR
1	¬ (negation)
2	∧ (and)
3	v (or)
4	$\rightarrow$ (conditional)
5	$\leftrightarrow$ (bi-conditional)

We usually use parenteses first, then use these rules of precedence to specify the order you apply the logical operators.

For Example:  $p \lor q \land r$  means  $p \lor (q \land r)$ rather than  $(p \lor q) \land r$ 

## **Truth tables**

In the truth tables below, F (or zero) denotes a false proposition while T (or one) denotes a true proposition.

p		NEGATION	CONJUNCTION	DISJUNCTION	CONDITIONAL	<b>BI-CONDITIONAL</b>	EXCLUSIVE OR
	q	₱ , (¬ p) not p	(p ㅅ q) p and q	(p v q) p or q	$(p \rightarrow q)$ If p, then q; p implies q; q whenever p;	(p ↔ q) p if and only if q	(p⊕q) p or q but not both
Т	Т	F	Т	Т	Т	Т	F
Т	F	F	F	Т	F	F	Т
F	Т	Т	F	Т	Т	F	Т
F	F	Т	F	F	Т	Т	F

р	q	$\begin{array}{c} \text{CONVERSE} \\ \textbf{q} \rightarrow \textbf{p} \end{array}$	$\begin{array}{c} \text{CONTRAPOSITIVE} \\ \overline{q} \rightarrow \overline{p} \end{array}$	$\begin{matrix} \text{INVERSE} \\ \overline{p} \rightarrow \overline{q} \end{matrix}$	
		lf q, then p	lf not q, then not p	lf not p, then not q	
Т	Т	Т	Т	Т	
Т	F	Т	F	Т	
F	Т	F	т	F	
F	F	Т	Т	Т	

Examples									
a) Construct a truth table for the compound proposition: $(p \rightarrow q) \leftrightarrow (\overline{q} \rightarrow \overline{p})$									
р	q	p	q	$p \rightarrow q$	$\overline{\mathbf{q}} \rightarrow \overline{\mathbf{p}}$	$(\mathbf{p}  ightarrow \mathbf{q}) \leftrightarrow (\mathbf{\overline{q}}  ightarrow \mathbf{\overline{p}})$			
Т	Т	F	F	Т	Т	Т			
Т	F	F	Т	F	F	Т			
F	Т	Т	F	Т	Т	Т			
F	F	Т	Т	Т	Т	Т			

b) Use a truth table to determine if these two compound propositions are equivalent:  $(p \rightarrow r) v (q \rightarrow r) and (p \land q) \rightarrow r$ 

р	q	r	$p \rightarrow r$	$q \rightarrow r$	p ^ d	$(p \rightarrow r) \vee (q \rightarrow r)$	$(p \land q) \rightarrow r$
Т	Т	Т	Т	Т	Т	Т	Т
Т	Т	F	F	F	Т	F	F
Т	F	Т	Т	Т	F	Т	Т
Т	F	F	F	Т	F	Т	Т
F	Т	Т	Т	Т	F	Т	Т
F	Т	F	Т	F	F	Т	Т
F	F	Т	Т	Т	F	Т	Т
F	F	F	Т	Т	F	Т	Т

Therefore, both propositions are equivalent as the truth values for both propositions result the same.

c) Use the following propositions to complete the questions below.

p: The sun is shining.

r: You wear sunglasses.

q: You will go to the beach.

s: You wear sunscreen.

i) Express the following compound propositions as symbols:

- 1) If the sun is not shining then you do not go to the beach.
- 2) If you go to the beach then you will wear sunscreen or sunglasses.
- 3) If the sun is shining and you are going to the beach then you will wear sunglasses and sunscreen.

ii) Express the following compound propositions as English sentences:

- 4)  $q \leftrightarrow p$
- 5)  $((\bar{p} \land q) \rightarrow \bar{r})$
- 6)  $p \rightarrow (s v r)$

#### Answers

#### i) Symbols

- 1.  $\bar{p} \rightarrow \bar{q}$
- 2.  $q \rightarrow (s v r)$
- 3.  $(p \land q) \rightarrow (r \land s)$

- ii) English
  - 4. You will go to the beach if and only if the sun is shining.
  - 5. If the sun is not shining and you go to the beach then you will not wear sunglasses.
  - 6. If the sun is shining then you will wear sunscreen or sunglasses.

### **Student Learning Centre**

Call: 905.721.8668 ext. 6578 Email: studentlearning@ontariotechu.ca Downtown Oshawa Location: Charles Hall. Website: ontariotechu.ca/studentlearning North Oshawa LocationShawenjigewining Hall

