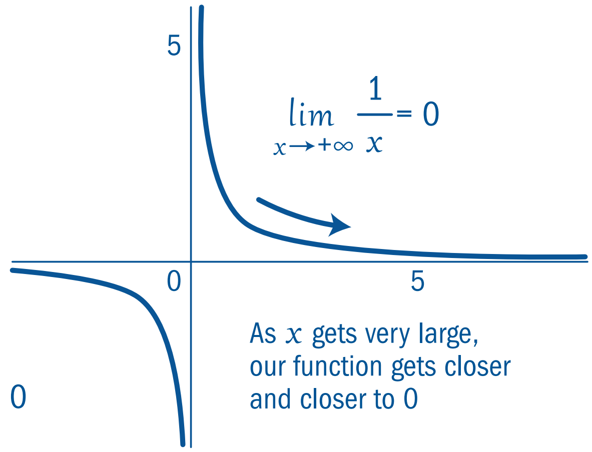
Introductory calculus

WHAT IS A LIMIT? $\lim_{x \to c} f(x) = L$

This means that as x **approaches** some value, c, the function **approaches** L, our limit. Limits help us understand how the function behaves.



As x gets very large, our function gets closer and closer to 0

LIMIT LAWS

$$\lim_{x \to c} (f(x) \pm g(x)) = L \pm M \qquad \lim_{x \to c} (f(x)\, g(x)) = L\, M$$

$$\lim_{x \to c} (kf(x)) = k \lim_{x \to c} (f(x)) = kL \qquad \lim_{x \to c} \left(\frac{f(x)}{g(x)} \right) = \frac{L}{M} \text{ if } M \neq 0$$

EVALUATING LIMITS

1. **Use direct substitution:** $\lim_{x \to c} f(x) = f(c)$
2. If this gives you the indeterminate form " $\frac{0}{0}$ " try factoring or rationalizing to simplify the expression first.

   Example:

$$\lim_{x \to -3} \frac{x^2 - 9}{x + 3} = \lim_{x \to -3} \frac{(x - 3)(x + 3)}{x + 3} = \lim_{x \to -3} x - 3 = -6$$

EVALUATING LIMITS AT INFINITY

We can't reach infinity but we want to see where our function is going as x gets very large.

Examples: $\lim_{x \to -\infty} 3x$ does not exist (D.N.E) since $3x$ will simply approach $-\infty$

For rational functions, $\frac{P(x)}{Q(x)}$:

$\lim_{x \to \infty} \frac{x^2 + 3}{x^4 - x} = 0$ since the degree of P is less than Q

$\lim_{x \to \infty} \frac{4x^3 + 5x}{7x^3 - 1} = \frac{4}{7}$ since the degree of P is equal to Q

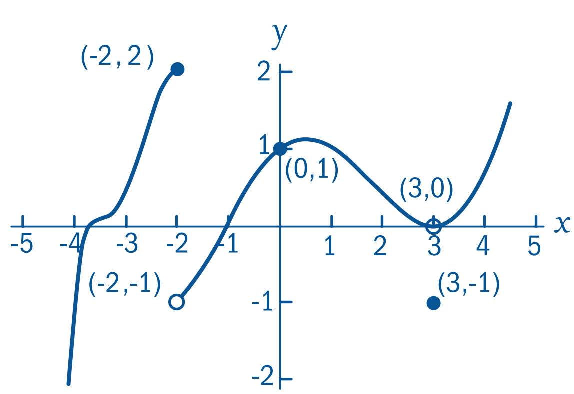
CONTINUITY

The function in the example is piecewise and clearly discontinuous. But to be **continuous at a specific point $x = c$** we need 3 conditions to be true:

1. $f(c)$ exists **2.** $\lim_{x \to c} f(x)$ exist **3.** $\lim_{x \to c} f(x) = f(c)$

On this graph, (0,1) is a continuous point, use the above condition to show why.

Example:



Does the limit exist?

The limit as we approach $x = -2$ from the left: $\lim_{x \to -2^-} f(x) = 2$

The limit as we approach $x = -2$ from the right: $\lim_{x \to -2^+} f(x) = -1$

$\lim_{x \to -2^-} f(x) \neq \lim_{x \to -2^+} f(x) \therefore \lim_{x \to -2} f(x)$ D.N.E.

DIFFERENTIATION

The derivative of a function $f(x)$ gives the slope of the tangent (instantaneous rate of change).

Previously we always used two points to find slope but the tangent only touches one point on the graph.

Secant slope: $\dfrac{f(x_1) - f(x_0)}{x_1 - x_0}$

To approximate the slope of the tangent we can take two points very close to each other, say h away, and calculate the slope of this secant.

Approximate Tangent Slope:

$$\frac{f(x_0 + h) - f(x_0)}{x_0 + h - x_0}$$

Now that we know about limits we can find the value of the slope of the tangent if we make h approach 0

So a derivative is defined as $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$.

DIFFERENTIATION RULES

Constant Rule: $f(x) = c$ then $f'(x) = \dfrac{df(x)}{dx} = 0$	$f(x) = 5 \qquad f'(x) = 0$
Power Rule: $\dfrac{d}{dx} x^n = nx^{n-1}$	$f(x) = x^3 \qquad f'(x) = 3x^2$
Constant Multiple Rule: $\dfrac{d}{dx}(cf(x)) = cf'(x)$	$f(x) = 3\sin x \qquad f'(x) = 3\cos x$
Sum Rule: $\dfrac{d}{dx}[f(x) \pm g(x)] = f'(x) + g'(x)$	$f(x) = 5\cos(x) - 2x^2$ $f'(x) = -5\sin(x) - 4x$
Product Rule: $\dfrac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$	$f(x) = (x^4 + x)e^{2x}$ $f'(x) = (4x^3 + 1)e^{2x} + (x^4 + x)(2e^{2x})$
Quotient Rule: $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$	$f(x) = \dfrac{\sin(x)}{2e^x} \qquad f'(x) = \dfrac{\cos(x)2e^x - \sin(x)2e^x}{4e^{2x}}$
Chain Rule: $\dfrac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$	$f(x) = \sqrt{3x^2 + 5} \qquad f'(x) = \dfrac{1}{2\sqrt{3x^2 + 5}}(6x)$

Useful Derivatives:

$\dfrac{d}{dx}(\sin x) = \cos x \qquad \dfrac{d}{dx}(\sec x) = \sec x \tan x \qquad \dfrac{d}{dx}(\ln x) = \dfrac{1}{x}$

$\dfrac{d}{dx}(\cos x) = -\sin x \qquad \dfrac{d}{dx}(\cot x) = -\csc^2 x \qquad \dfrac{d}{dx}(a^x) = (\ln a)a^x$

$\dfrac{d}{dx}(\tan x) = \sec^2 x \qquad \dfrac{d}{dx}(\csc x) = -\csc x \cot x \qquad \dfrac{d}{dx}(e^x) = e^x$

FOR MORE INFORMATION OR TO BOOK AN APPOINTMENT

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