Calculus Refresher: Integration by Substitution

The following guideline summarizes the steps involved in u-substitution method:

Integration by substitution

Choose a new variable u. Usually try choosing u to be some complicated part of the integrand whose derivative is also in the integrand.

Compute du.

Replace all terms in the original integrand with expressions involving u and du.

Evaluate the resulting u integrand. (If you can't, you may need to try a different u or a different method of integration.)

Replace u with the corresponding expression in x.

Example: Evaluate $\int \frac{x}{x^2+2}\,dx$

We would choose $u = x^2 + 2$ since its derivative is $2x$ and we can write the entire expression in terms of u and du.

$$u = x^2 + 2$$

$$du = 2x\,dx \longrightarrow \frac{du}{2} = x\,dx$$

When substituting we should get the entire expression in terms of u and du.

$$\int \frac{x}{x^2+2}\,dx = \int \frac{\frac{du}{2}}{u}$$

$$= \frac{1}{2}\int \frac{du}{u}$$

$$= \frac{1}{2}\ln|u| + C$$

$$= \frac{1}{2}\ln(x^2+2) + C$$

Now you try it!
Evaluate:

$$\int x^3\sqrt{x^4+5}\,dx$$

Answer: $\frac{(x^4+5)^{3/2}}{6} + C$

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