Factoring Polynomials

Factoring is finding a product of polynomials which is equivalent to a given expression.

Example:

$$8x^2-28x^2=4x^2(2x-7)$$

Product

$$x^2-2x-3=(x+1)(x-3)$$

Product

Factoring Methods

1. Greatest Common Factor
2. Using Formulas
3. Factoring by Grouping

1. GREATEST COMMON FACTOR

If all terms of a polynomial share a common factor, then the first step is to factor out the greatest common factor.

Example: $8x^2-12x$

Both $8x^2 - 12x$ have a common factor of $4x$.

$$8x^2=4x * 2x$$

$$12x=4x * 3$$

Factoring out $4x$,

$$8x^2 - 12x = 4x(2x - 3)$$

Practice:

1. Factor each expression.

a) $-3p^4 - 9p^2 - 6p^3$ b) $21z^2 + 56$ c) $-6x^2y^3 + 12x^4y - 9x^3y^5$

2. FACTORING USING FORMULAS

The following formulas can be used for factoring special polynomials that contain two terms:

Difference of Squares $a^2 - b^2 = (a-b)(a+b)$

Difference of Cubes $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Sum of Cubes $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

Note: The sum of two perfect squares a^2+b^2 **cannot** be factored using real numbers.

Example: Factor $25x^2 - 9$

Since $25x^2$ and 9 are perfect squares, the given expression is a difference of squares and can be written as $(5x)^2 - 3^2$.

Using the difference of squares formula,

$$25x^2 - 9 = (5x - 3)(5x + 3)$$

Practice:

2. Factor each expression.

a) $4x^2 - 9$

b) $8x^3 - 125$

c) $28n^3 - 7n$

d) $16z^4 - 81$

e) $x^2 - \dfrac{1}{16}$

3. FACTORING BY GROUPING

If a polynomial has four or more terms, it may be necessary to group terms in such a way where each group shares a common factor.

Example: Factor $9x^3 + 6x^2 + 15x + 10$

It may appear this polynomial is not factorable, since there is no common factor for all four terms. However, grouping the terms and factoring the first two terms and the second two terms separately reveals a common factor:

$$\underbrace{9x^3+6x^2}+\underbrace{15x + 10} = 3x^2\underbrace{(3x + 2)} + 5\underbrace{(3x + 2)}$$

Common Factor

Factoring is completed when the common factor (3x+2) is factored out to give the following:

$$\begin{aligned} 9x^3+6x^2+15x + 10 &= 3x^2(3x + 2) + 5(3x + 2) \\ &= (3x + 2)(3x^2 + 5) \end{aligned}$$

FACTORING QUADRATIC POLYNOMIALS BY GROUPING

To factor a quadratic polynomial of the form

$$ax^2 + bx + c$$

the middle term needs to be split using two numbers which multiply to give ***ac*** and which add to give ***b***. This polynomial with four terms can then be factored by grouping.

Example: Factor $6x^2 + 5x - 4$

To split the middle term, two numbers are needed such that their product is ***ac*** and the sum is ***b***.

Since ***ac*** = (6) (- 4) = - 24 and ***b*** = 5, the two number are 8 and -3.

Check: $(8)\,(-3) = -24$

$8 + (-3) = 5$

The factorization is completed by using these two numbers to split the middle term and then factoring by grouping:

$$\begin{aligned} 6x^2+5x - 4 &= 6x^2 + \mathbf{8x} - \mathbf{3x} - 4 \\ &= 2x(3x + 4) - 1(3x + 4) \\ &= (3x + 4)(2x - 1) \end{aligned}$$

Practice:

3. Factor each expression completely, if possible.

a) $x^2 + 7x + 10$

b) $k^5 - 3k^4 - 9k + 27$

c) $2b^2 - 10b - 28$

d) $12x^2 - 40x + 4x^3$

e) $3x^2 + 10x + 8$

f) $3x^2 - xy - 6x + 2y$

ANSWERS:

1. a) $-3p^2(p^2 + 2p + 3)$
   b) $7(3z^2 + 8)$
   c) $3x^2y(4x^2 - 3xy^4 - 2y^2)$

2. a) $(2x - 3)(2x + 3)$
   b) $(2x - 5)(4x^2 + 10x + 25)$
   c) $7n(2n - 1)(2n + 1)$
   d) $(2z - 3)(2z + 3)(4z^2 + 9)$
   e) $(x - \frac{1}{4})(x + \frac{1}{4})$

3. a) $(x + 5)(x + 2)$
   b) $(k - 3)(k^2 - 3)(k^2 + 3)$ or $(k - 3)(k - \sqrt{3})(k + \sqrt{3})(k^2 + 3)$
   c) $2(b + 2)(b - 7)$
   d) $4x(x + 5)(x - 2)$
   e) $(3x + 4)(x + 2)$
   f) $(x - 2)(3x - y)$