Useful Counting Rules for Probabilities

The probability of an event is given by $P(\text{event}) = \dfrac{\text{number of outcomes in event}}{\text{total number of outcomes}}$

However, at times it can be tricky to find the number of outcomes. To assist us with this task, here is a table which summarizes a few simple rules.

How the experiment is performed	Total number of outcomes to perform the experiment	Example
The multi-stage experiment: The experiment has $k1$ ways to perform the first stage, $k2$ ways to perform the second stage, $k3$ ways to perform the third stage, etc., and there are r such stages	$k_1 k_2 k_3 \ldots k_r$	**a)** Two dice are tossed. The total number of outcomes is (6)(6)=36. **b)** You own 4 pairs of jeans, 12 T-shirts, and 4 pairs of sneakers. The total number of outfits is (4)(12)(4)=192
Orderings or Permutations: (*order matters; no item is used more than once*) Arranging r objects from a group of n Arranging n objects from a group of n	$\dfrac{n!}{(n-r)!}$ $n!$	**a)** A committee of eight people must choose a president, a vice-president, and a secretary. The number of ways this can be done is $\dfrac{8!}{(8-3)!} = (8)(7)(6) = 336$ **b)** You want to visit six cities. The number of different trips possible is 6!=(6)(5)(4)(3)(2)(1)=720
Combinations : (*order does not matter; no item is used more than once*) Choosing r objects from a group of n	$\binom{n}{r} = \dfrac{n!}{r!(n-r)!}$ n choose r	Four cards are selected from a 52-card deck. The number of ways of selecting four cards is $\dfrac{52!}{4!(52-4)!} = 270{,}725$

Now, let's use these rules to solve some probability problems.

Example 1: A computer system requires that passwords contain at least one digit. If four characters are generated at random, and each is equally likely to be any of the 26 letters or 10 digits, what is the probability that a valid password will be generated?

Solution: The probability of generating a proper password with at least one digit is

$$P(\text{at least one digit}) = \frac{\#\text{ of ways to choose 4 characters with at least one digit}}{\text{total }\#\text{ of possible outcomes}}$$

$$= \frac{(\#\text{ of ways to choose 4 characters}) - (\#\text{ of ways to choose 4 characters with no digit})}{\text{total }\#\text{ of possible outcomes}}$$

$$= \frac{(36)(36)(36)(36) - (26)(26)(26)(26)}{(36)(36)(36)(36)} = \frac{36^4 - 26^4}{36^4} = 0.73$$

Example 2: Suppose the group of twelve consists of five men and seven women. Find the probability that the selected five-person group consists of

a) three men **b)** one man and two women **c)** at least one man

Solution:

a) The probability of a five-person group to consist of three men is

$$P(3\text{ men}) = \frac{\#\text{ of ways to choose 3 men}}{\text{total }\#\text{ of possible outcomes}} = \frac{\binom{5}{3}}{\binom{12}{5}} = \frac{\frac{5!}{(5-3)!3!}}{\frac{12!}{(12-5)!5!}} = \frac{10}{792} = 0.01$$

b) The probability of selecting one man and two women in a five-person group is

$$P(1\text{ man and 2 women}) = \frac{\#\text{ of ways to choose 1 man and 2 women}}{\text{total }\#\text{ of possible outcomes}}$$

$$= \frac{(\#\text{ of ways to choose 1 man})(\#\text{ of ways to choose 2 women})}{\text{total }\#\text{ of possible outcomes}}$$

$$= \frac{\binom{5}{1}\binom{7}{2}}{792} = \frac{\left(\frac{5!}{(5-1)!1!}\right)\left(\frac{7!}{(7-2)!2!}\right)}{792} = \frac{(5)(21)}{792} = 0.13$$

c) The probability of selecting five people with at least one man is

$$P(\text{at least 1 man}) = \frac{(\text{total }\#\text{ of ways to choose 5 people}) - (\#\text{ of ways to choose a group with no man})}{\text{total }\#\text{ of possible outcomes}}$$

$$= \frac{\binom{12}{5} - \binom{7}{5}}{792} = \frac{792 - 21}{792} = \frac{771}{792} = 0.97$$

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