Circular Motion

WHAT IS CIRCULAR MOTION?

Circular motion refers to an object undergoing motion in a circular path that can be specified by a radius, r, and angular position, θ. The study of circular motion is referred to as the kinematics of circular motion.

The distance traveled by an object undergoing circular motion is defined as:

$$s = \theta r$$

The **angular velocity**, ω, of an object is defined as the derivative of the angular position with respect to time:

$$\omega = \frac{d\theta}{dt}$$

The **angle acceleration**, α, is defined as the derivative of the angular velocity with respect to time:

$$\alpha = \frac{d\omega}{dt}$$

LINEAR AND CIRCULAR MOTION RELATIONSHIPS

Linear motions and circular motions are connected, and they can be converted from one to another. The linear velocity, v, of the object is always tangential to the circular direction and has a magnitude of:

$$v = \omega r$$

The linear acceleration of the object is decomposed into radial and tangential acceleration. The radial acceleration is defined as:

$$a_r = \frac{v^2}{r} = \omega^2 r$$

and is pointing towards the center of the circle. While the tangential acceleration is defined as:

$$a_t = \alpha r$$

and acts perpendicular to the radial direction. The total acceleration is defined as:

$$a = \sqrt{a_r^2 + a_t^2}$$

CONSTANT ANGULAR ACCELERATION

If the object's motion is described with constant angular acceleration, the following equations are valid and can be used:

$$\theta_f = \theta_i + \omega_i \, \Delta t + \frac{1}{2} \alpha \, \Delta t^2$$

$$\omega_f = \omega_i + \alpha \, \Delta t$$

$$\omega_f^2 = \omega_i^2 + 2 \, \alpha \, \Delta \theta$$

SAMPLE PROBLEMS

1. A 900-kg car moving at 10 m/s drives around a circle with a radius of 25.0 m. Determine the acceleration and the net force acting upon the car. [4 m/s2, 3600N]

2. A 95-kg halfback makes a turn on the football field. The halfback sweeps out a path that is a portion of a circle with a radius of 12-meters. The halfback makes a quarter of a turn around the circle in 2.1 seconds. Determine the speed, acceleration and net force acting upon the halfback. [9 m/s, 6.7 m/s2, 637 N]

3. The Wall of Death is a circus act in which cars and motorcycles drive along the inside of a vertical cylinder/silo as shown in the figure below. The cars are capable of travelling at a maximum of 30.0 m/s while in the cylinder. Coefficients of friction between rubber and concrete are given as $\mu_s = 1.00$ and $\mu_k = 0.800$. What is the maximum diameter of the cylinder/silo so that cars can safely travel around the cylinder without drivers falling to their deaths? At this speed, how long does it take a car to travel ten revolutions within the cylinder? [184 m; 192 s]