

# Calculating Probabilities

Assuming equally likely outcomes, the **probability** of an event can be calculated as

$$p(\text{event}) = \frac{\text{number of outcomes in event}}{\text{total number of outcomes}}$$

**Example:** The following table summarizes the numbers of defective and nondefective medical devices produced by two plants:

	Plant A	Plant B
Defective	20	30
Nondefective	180	70

a) The probability of selecting a defective device is

$$p(\text{defective}) = \frac{\text{Number of defective devices}}{\text{Total number of devices}} = \frac{20 + 30}{20 + 30 + 180 + 70} = \frac{50}{300} = 0.17$$

b) The probability of selecting a device produced by plant B is

$$p(\text{plant B}) = \frac{\text{Number of devices from plant B}}{\text{Total number of devices}} = \frac{30 + 70}{300} = \frac{100}{300} = 0.33$$

c) The probability of selecting a defective device from plant B

$$p(\text{defective} \cap \text{plant B}) = \frac{\text{Number of defective devices that are from plant B}}{\text{Total number of devices}} = \frac{30}{300} = 0.1$$

## Some useful Probability Properties

The probability of an event A not occurring is given by

**NOT A**  $P(A') = 1 - P(A)$

The probability of either event A or event B occurring is given by

**A or B**  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

If the events A and B are mutually exclusive (cannot both occur at the same time), then

$$P(A \cap B) = 0 \quad \text{and} \quad P(A \cup B) = P(A) + P(B)$$

**Example:** Being married and being over 30 years old are **not mutually exclusive** events.  
Being a teenager and being over 30 years old **are mutually exclusive** events.

The **conditional probability** of event A, given that event B has occurred is

**A given B**  $P(A | B) = \frac{P(A \cap B)}{P(B)}$  or rearranging  $P(A \cap B) = P(A | B)P(B)$

### Remember:

The sum of the probabilities of all possible outcomes is 1

The probability of each outcome is a number between 0 and 1

If two events A and B are **independent** (the occurrence of one event does not influence the probability of the other), then

$$P(A | B) = P(A) \text{ and } P(A \cap B) = P(A)P(B)$$

**Example:** Consider the table below with the probabilities as shown:

	Male	Female
Colourblind	0.04	0.002
Not colourblind	0.47	0.488

**Find:**

a) The probability of selecting a colourblind male

$$P(\text{Male} \cap \text{colourblind}) = 0.04 \text{ (Taken directly from the table)}$$

b) The probability of selecting a non colourblind female

$$P(\text{Not colourblind} \cap \text{female}) = 0.488 \text{ (Taken directly from the table)}$$

c) The probability of selecting a male

$$0.04 + 0.47 = 0.51$$

**Explanation:** A male is colourblind **or** not colour blind and since events 'colourblind' and 'not colourblind' are mutually exclusive, we can use  $P(A \cup B) = P(A) + P(B)$

d) The probability of selecting a colourblind person

$$0.04 + 0.002 = 0.042$$

e) The probability of selecting either a male **or** a colourblind person

$$\begin{aligned} P(\text{male} \cup \text{colourblind}) &= P(\text{male}) + P(\text{colourblind}) - P(\text{male} \cap \text{colourblind}) \\ &= 0.51 + 0.042 - 0.04 \\ &= 0.512 \end{aligned}$$

f) The probability of being colourblind, **if** the person is male:

↑  
Given

$$\begin{aligned} P(\text{colourblind} | \text{male}) &= \frac{P(\text{colourblind} \cap \text{male})}{P(\text{male})} \\ &= \frac{0.04}{0.51} \\ &= 0.078 \end{aligned}$$

g) Determine if two events 'being a male' and 'being colourblind' are independent.

If two events are **independent**, then  $P(A \cap B) = P(A)P(B)$

$$P(\text{colourblind} \cap \text{male}) = 0.04$$

$$P(\text{colourblind})P(\text{male}) = (0.042)(0.51) = 0.02$$

Not equal; therefore, dependent

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