Beyond 48 Hours!

Leanna Calla, Dr Sean Bohun & Dr Helene LeBlanc

Faculty of Science, University of Ontario Institute of Technology
leanna.calla@uoit.net, sean.bohun@uoit.ca, helene.leblanc@uoit.ca

Introduction

Within 48 hours of death, forensic investigators use pathological evidence to determine time of death. Beyond 48 hours, forensic investigators rely on the work of entomologists to give informed estimations of the post mortem interval (PMI). Under the supervision of a mathematics professor, Dr. Sean Bohun, and a forensic science professor, Dr. Helene LeBlanc, we have identified an area of the current entomological methods that can be modelled mathematically to improve accuracy of time of death estimations.

Current Methods

The current methods carried out by entomologists after a body is discovered is a multi-step process. Listed below are the steps followed by entomologists to give an informed estimation of PMI.

1. Primary colonizing species are sampled from the scene. Ideally two samples of each species type, one for preservation and one for rearing.
2. A data logger is left at the scene in a similar state as the body, i.e sun/shade exposure, under a tarp, etc.
3. In the lab, the first sample is preserved and the approximate location in the life cycle is recorded.

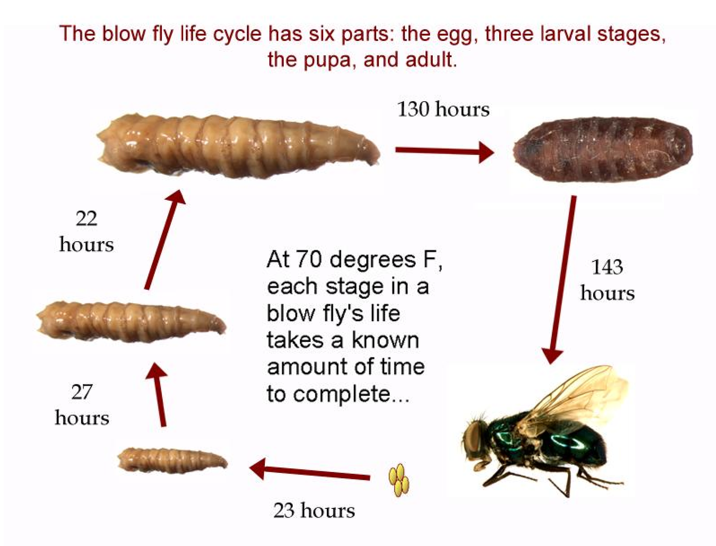

Fig. 1: Life cycle of a blow fly

4. The second sample is reared to adulthood in a temperature controlled area. This is done to aid in the identification of the species and confirm the location in the life cycle when sampled.
5. Daily average temperature data is obtained from Environment Canada weather stations that are in the vicinity of scene.
6. Linear regression is performed between the data logger data and the Environment Canada data to predict the temperature at the scene before the body was found.

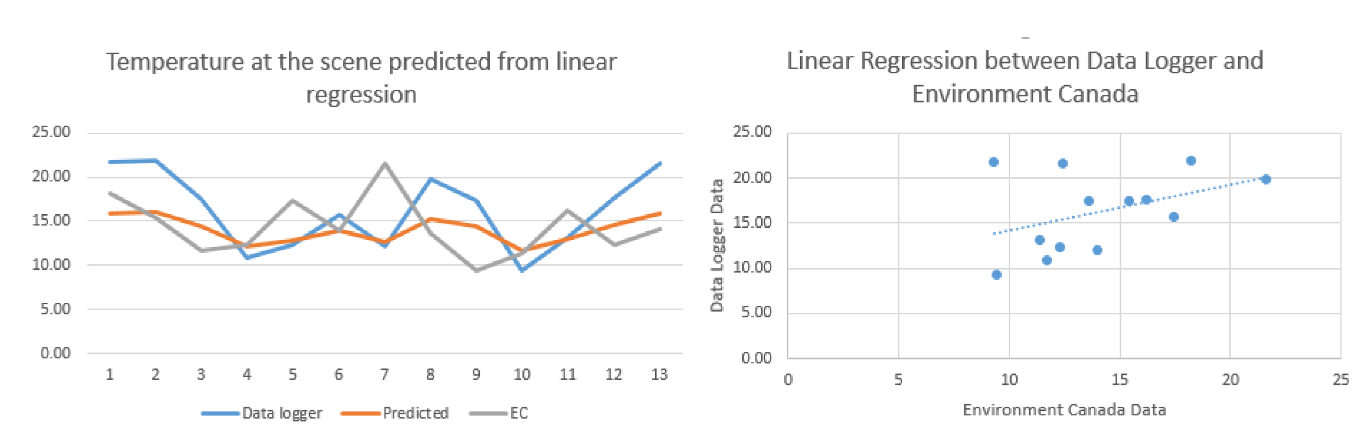

Fig. 2: L: Daily average temperatures for the 13 days when the data logger is present at the scene. R: Linear regression on the daily averages results in a coefficient of determination of 0.1726

7. Accumulated degree days, or accumulated degree hours, are used to predict back when eggs may have been laid. This time interval is used as an estimate for PMI.

Modelling

A key ingredient of the current method is an accurate determination of temperature in the vicinity of the body. To increase the resolution we propose a model for each hour that evolves according to convective and radiative effects with coupling that changes slowly during any 24 hour period. In detail, for hour $j \in \{1, \ldots, 24\}$ the data logger temperature varies according to

$$\frac{\mathrm{d}T^j}{\mathrm{d}t} = c_0^j(T_0^j - T^j) + c_1^j({T_0^j}^4 - {T^j}^4) \qquad (1)$$

where T^j is the temperature of the data logger, t is time, T_0^j is the estimate of the local temperature and $c_0^j > 0$, $c_1^j > 0$ are slowly varying coefficients modelling convective and radiative heat exchange respectively. The idea is to specify the initial and final temperatures of the data logger for each hour and then find a sequence of hourly local environmental temperatures $\{T_0^j\}$ so that:

- the daily variation of $\{c_0^j\}$ and $\{c_0^j\}$ is minimized;
- the average of the $\{T_0^j\}$ matches the given daily average temperature that was supplied by Environment Canada.

This model produces daily profiles of $\{c_0^j\}$, $\{c_1^j\}$ and $\{T_0^j\}$ that will characterize the environment which can then be used to infer a most likely environmental temperature behaviour into the past from the given temperature averages. The location details are encoded in the daily characteristic behaviour of $\{c_0^j\}$ and $\{c_1^j\}$.

The model was tested against case study data, in which we have higher resolution data logger samples (hourly) and lower resolution Environment Canada samples (daily). Figure 3 shows $T(t)$ over two successive hours with the temperature cooling and then heating. For each hour the value of $c_0^j > 0$ naturally restricts the possible values of T_0^j that ensure $c_1^j > 0$. This dependency is depicted in the subplot on the right hand side.

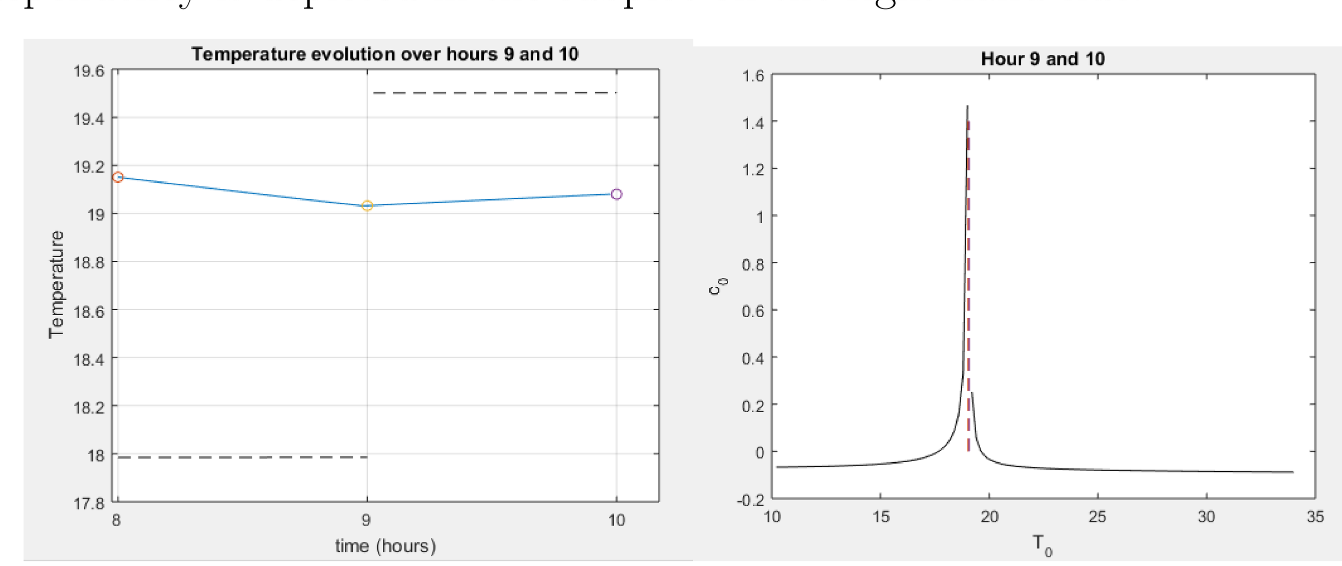

Fig. 3: L: $T(t)$ for two consecutive hours. Dashed lines mark the required T_0 values and the circles are the observed data logger values. R: Relationship between c_0, c_1 and T_0. The admissible domain of c_1 is illustrated by the two curves: $c_1 = 0$ and $c_1 \to \infty$. The domain shrinks with increasing c_0

Ideally, we assume that c_0 and c_1 will be slow evolving quantities throughout the day with the small variations attributed to significant changes in weather. Towards this goal, the specialized cases we considered are:

P_1: Let c_1 remain constant. Find c_0^+, c_0^- such that $\|c_0^+ - c_0^-\|_{\ell^1}$ is minimized

P_2: Let c_0 remain constant. Find c_1^+, c_1^- such that $\|c_1^+ - c_1^-\|_{\ell^1}$ is minimized

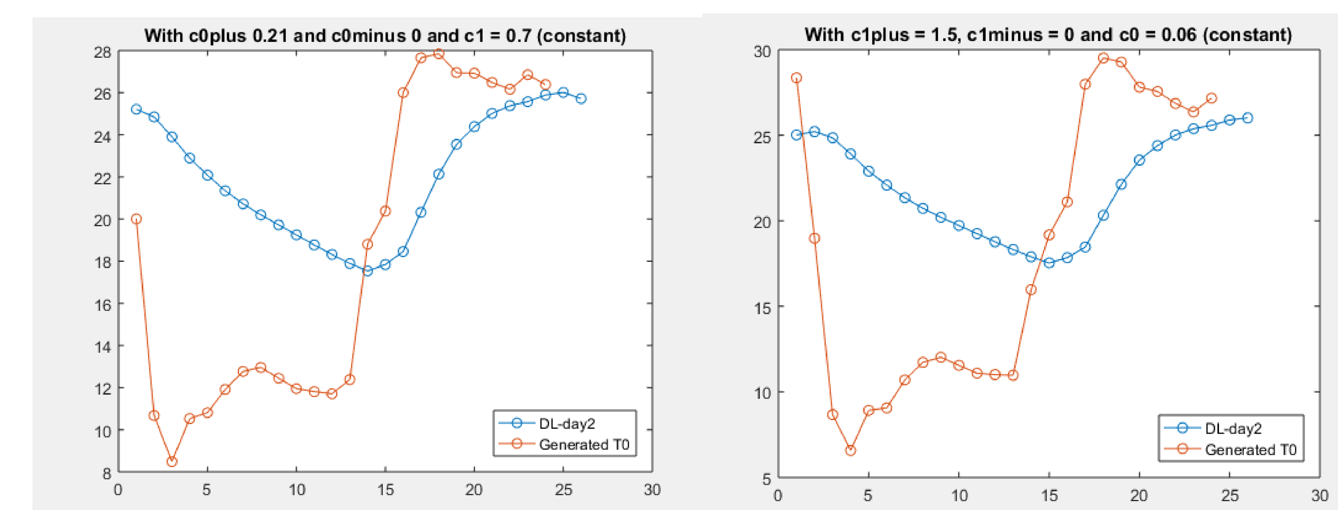

Fig. 4: L: Solution to P_1. Radiative heat exchange coefficient is held constant, convective coefficient has one transition, $\|c_0^+ - c_0^-\|_{\ell^1} = 0.21$. R: Solution to P_2. Convective heat exchange coefficient is held constant, radiative coefficient has one transition, $\|c_1^+ - c_1^-\|_{\ell^1} = 1.5$

During the implementation of finding a solution, variations of c_0^j, the convective coefficient, quickly restrict the range of of possible T_0^j values. This is indicative of c_1^j, the radiative coefficient, requiring more variation to satisfy the average temperature constraint.

Future Work

If the model is a sufficient temperature predictor with high accuracy, it can be used by entomologists to improve time of death estimations. Further research is required before the model is ready to be used by entomologists. For example, efficient algorithms for minimizing with respect to ℓ^1 norms can be explored, also the model can be tested against other case studies to search for patterns in c_0 and c_1 based on time of year, daily weather patterns or geographic location.

A natural extension of this project is the development of a mathematical model for the growth cycle of a decompositional species. A modernized maggot growth model could be created and would be very helpful in precisely determining bounds on the age of the sampled species. We believe that an accurate model to predict past temperatures at a crime scene that naturally reflects the local microclimate is a crucial step in obtaining a precise estimation for time of death.

Acknowledgements

This research was supported in part by an NSERC (Natural Sciences and Engineering Research Council of Canada) USRA (Undergraduate Student Research Award). LC and SB would like to thank HB for providing the environmental datasets.